SYLLABUS

FOR

M.SC. IN MATHEMATICS
(UNDER CBCS SYSTEM)

A FOUR SEMESTERS COURSE

(Effective from the academic session 2020 – 2021 and onwards)

COOCH BEHAR PANCHANAN BARMA UNIVERSITY

COOCH BEHAR, WEST BENGAL

COOCH BEHAR PANCHANAN BARMA UNIVERSITY

Syllabus for M.Sc. in Mathematics
(Under CBCS system)

OBJECTIVE:

The duration of the Post Graduate course in Mathematics of Cooch Behar Panchanan Barma University will consist of two years with Semester-I, Semester-II, Semester-III and Semester-IV each of six months duration leading to Semester-I, Semester-II, Semester-III and Semester-IV examinations in Mathematics at the end of each semester.

Syllabus for the P.G. course in Mathematics under CBCS system is hereby framed following the guidelines of U.G.C. according to the following schemes and structures. All the students admitted to P.G. course in Mathematics shall take courses of Semester-I, Semester-II, Semester-III and Semester-IV.

SCHEME:

Total Marks = 1600 with 400 marks in each semester comprising of four papers in each semester with 100 marks in each paper. Out of the total marks 20% marks is allotted for Continuous Evaluation/Class test and 5% marks is allotted for regular attendance. In Semester-I, four core courses will be taught. In a similar manner, another four core courses will be taught in Semester-II.

All the written papers will be evaluated by internal examiners only. The Internal Assessment Tests (Continuous Evaluations) will be taken by the Department and all the internal members will evaluate the answer scripts on the respective papers/topics.

SEMESTER COURSE STRUCTURE OF M.Sc. (MATHEMATICS) under CBCS system

SYLLABUS for SEMESTERS – I and II

SEMESTERS	PAPERS	TOPICS	MARKS (CREDIT)
Semester I	Core -1	Real Analysis	100 (5)
	Core - 2	Abstract Algebra	100 (5)
	Core - 3	Ordinary Differential Equations and Special Functions	100 (5)
	Core - 4	Classical Mechanics	100 (5)
Semester II	Core - 5	Complex Analysis	100 (5)
	Core - 6	Linear Algebra	100 (5)
	Core - 7	Partial Differential Equations	100 (5)
	Core - 8	Continuum Mechanics	100 (5)

PAPER WISE DISTRIBUTION OF MARKS

Sl. No.	Paper	Written	Class Test	Attendance	Total	Credit
1.	Core - 1	75	20	05	100	05
2.	Core - 2	75	20	05	100	05
3.	Core - 3	75	20	05	100	05
4.	Core - 4	75	20	05	100	05
5.	Core - 5	75	20	05	100	05
6.	Core - 6	75	20	05	100	05
7.	Core - 7	75	20	05	100	05
8.	Core - 8	75	20	05	100	05

SEMESTER I

Duration: 6 Months (Including Examinations)
Total Marks: 400, Total No. of Lectures: 70 (70 Hours per paper)

	Papers	Topics	Marks (Credit)
Semester I	Core - 1	Real Analysis	100 (5)
	Core - 2	Abstract Algebra	100 (5)
	Core - 3	Ordinary Differential Equations and Special Functions	100 (5)
	Core - 4	Classical Mechanics	100 (5)

Core - 1
REAL ANALYSIS
(70 LECTURES)

Bounded Variation:

Functions of Bounded Variation and their properties, Riemann- Stieltjes integrals and its properties, Absolutely Continuous Functions.

The Lebesgue Measure:

Lebesgue Measure: (Lebesgue) Outer measure and measure on R, Measurable sets form an σ-algebra, Borel sets, Borel σ - algebra, open sets, closed sets are measurable, Existence of a non-measurable set, Measure space, Measurable Function and its properties, Borel measurable functions, Concept of Almost Everywhere (a.e.), sets of measure zero, Steinhaus Theorem, Sequence of measurable functions, Egorov's Theorem, Applications of Lusin Theorem.

The Lebesgue Integral:

Simple and Step Functions, Lebesgue integral of simple and step functions, Lebesgue integral of a bounded function over a set of finite measure, Bounded Convergence Theorem, Lebesgue integral of non-negative function, Fatou's Lemma, Monotone Convergence Theorem. The General Lebesgue integral: Lebesgue Integral of an arbitrary Measurable Function, Lebesgue Integrable functions. Dominated Convergence Theorem. Convergence in Measure. Riemann Integral as Lebesgue Integral. Product measure spaces, Fubini's Theorem (applications only).

References:

1. Apostol, T.M., Mathematical Analysis, Narosa Publishing House, 2002.
2. Royden, H.L., Fitzpatrick P.M., Real Analysis, 4th Edition, Pearson.
3. Aliprantis, C.D., Burkinshaw, O., Principles of Real Analysis, Third Edition, Harcourt Asia Pvt. Ltd., 1998.

Further Reading:

1. Halmos, P.R., Measure Theory, Springer, 2007.
2. Rudin, W., Principles of Mathematical Analysis, Tata McGraw Hill, 2001.
3. Rudin, W., Real and Complex Analysis, McGraw-Hill Book Co., 1966.
4. Tao, T., An Introduction to Measure Theory, American Mathematical Society.
5. Kolmogorov, A.N., Fomin, S.V., Measures, Lebesgue Integrals, and Hilbert Space, Academic Press,
6. New York & London, 1961.
7. Rana, I.K., An introduction to Measure and Integration, Second Edition, Narosa.
8. Barra, G.D., Measure Theory and Integration, Woodhead Pub.
9. Kingman, J.F.C. and Taylor, S.J., Introduction to Measure and Probability, Cambridge University Press, 1966.
10. Cohn, D.L., Measure Theory, Birkhauser, 2013.
11. Wheeden, R.L. and Zygmund, A., Measure and Integral, Monographs and Textbooks in Pure and Applied Mathematics, 1977.
12. Sohrab, H.H., Basic Real Analysis, Birkhauser, 2003.

Core - 2

ABSTRACT ALGEBRA

(70 LECTURES)

Groups:

Review of basic concepts of Group Theory: Lagrange's Theorem, Cyclic Groups, Permutation Groups and Groups of Symmetry: S_n; A_n; D_n, Conjugacy Classes, Index of a Subgroup, Divisible Abelian Groups. Homomorphism of Groups, Normal Subgroups, Quotient Groups, Isomorphism Theorems, Cayley's Theorem.

Direct Product and Semi-Direct Product of Groups, Fundamental Theorem (Structure Theorem) of Finite Abelian Groups, Cauchy's Theorem, Group Action, Sylow Theorems and their applications. Solvable Groups (Definition and Examples only). Field extension, Galois' theory. Modules.

Rings:
Ideals and Homomorphisms, Prime and Maximal Ideals, Quotient Field of an Integral Domain, Polynomial and Power Series Rings. Divisibility Theory : Euclidean Domain, Principal Ideal Domain, Unique Factorization Domain, Gauss' Theorem, Irrudicibility of polynomials, Chinese remainder theorem.

References:

1. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.
2. Gallian, J., Contemporary Abstract Algebra, Narosa, 2011.
3. Herstein, I.N., Topics in Abstract Algebra, Wiley Eastern Limited.
4. Sen M.K., Ghosh S., Mukhopadhyay P., Topics in Abstract Algebra, Universities Press.

Further Reading:

1. Roman, S., Fundamentals of Group Theory: An Advanced Approach, Birkhauser, 2012.
2. Malik, D.S., Mordesen, J.M., Sen, M.K., Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc, 1997.
3. Rotman, J., The Theory of Groups: An Introduction, Allyn and Bacon, Inc., Boston.
4. Rotman, J., A First Course In Abstract Algebra, Prentice Hall, 2005.
5. Pinter, Charles. C., A Book of Abstract Algebra, McGraw Hill, 1982.
6. Fraleigh, J.B., A First Course in Abstract Algebra, Narosa.
7. Jacobson, N., Basic Algebra, I & II, Hindusthan Publishing Corporation, India.
8. Hungerford, T.W., Algebra, Springer.
9. Artin, M., Algebra, Prentice Hall of India, 2007.
10. Goldhaber, J.K., Ehrlich, G., Algebra, The Macmillan Company, Collier-Macmillan Limited, London.
11. Gopalakrishnan, N.S., University Algebra, New Age International, 2005.

Core - 3

ORDINARY DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS
(70 LECTURES)

Ordinary Differential Equations:

Existence and Uniqueness:
First order ODE, Initial value problems, Existence theorem, Uniqueness, basic theorems, Ascoli Arzela theorem (statement only), Theorem on convergence of solution of initial value problems. Picard – Lindelöf theorem (statement only), Peano's existence theorem (statement only) and corollaries.

Boundary Value Problems for Second Order Equations:
Ordinary Differential Equations of the Strum-Liouville type and their properties, Application to Boundary Value Problems, Eigenvalues and Eigenfunctions, Orthogonality theorem, Expansion theorem. Green's function for Ordinary Differential Equations, Application to Boundary Value Problems.

Special Functions:

Singularities:
Fundamental System of Integrals, Singularity of a Linear Differential Equation. Solution in the neighbourhood of a singularity, Regular Integral, Equation of Fuchsian type, Series solution by Frobenius method.

Legendre Polynomials:
Legendre Functions, Generating Function, Legendre Functions of First & Second kind, Laplace Integral, Orthogonal Properties of Legendre Polynomials, Rodrigue's Formula.

Bessel Functions:
Bessel's Functions, Series Solution, Generating Function, Integral Representation of Bessel's Functions, Recurrence Relations, Asymptotic Expansion of Bessel Functions.

Hermite Polynomial:
Hermite equation and its solution, Generating function, Rodrigue's formula, Recurrence relations, Orthogonal Properties of Hermite Polynomials.

Lagurre polynomial:
Lagurre equation and its solution, Generating function, Recurrence relations, Orthogonal Properties of Hermite Polynomials.

Hypergeometric Function:
Hypergeometric Functions, Series Solution near zero, one and infinity. Integral Formula, Confluent Hypergeometric function, Integral representation of Hypergeometric function, Differentiation of Hypergeometric Function.

References:

1. Simmons, G.F., Differential Equations, Tata McGraw Hill.
2. Agarwal, Ravi P. and O' Regan D., An Introduction to Ordinary Differential Equations, Springer, 2000.

Further Reading:

1. Codington, E.A and Levinson, N., Theory of Ordinary Differential Equation, McGraw Hill.
2. Ince, E.L., Ordinary Differential Equation, Dover.
3. Estham, M.S.P., Theory of Ordinary Differential Equations, Van Nostrand Reinhold Compa.Ny, 1970.

4. Piaggio, H.T.H., An Elementary Treatise On Differential Equations And Their Applications, G. Bell And Sons, Ltd, 1949.
5. Hartman, P., Ordinary Differential Equations, SIAM, 2002.
6. Zill, D. G., Cullen, M.R., Differential Equations with Boundary Value Problems, Brooks/Cole, 2009.

Core - 4

CLASSICAL MECHANICS

(70 LECTURES)

Dynamical systems, Generalized coordinates, Degrees of freedom, Principle of virtual work. D'Alembert's principle. Unilateral and bilateral constraints. Holonomic and non-holonomic system. Lagrange's equations for holonomic systems. Lagrange's equation for impulsive forces and for systems involving dissipatative forces. Conservation theorems. Hamilton's principle and principle of least action. Hamilton's canonical equations. Canonical transformation with different generating functions. Lagrange and Poisson brackets and their properties. Hamilton-Jacobi equations and separation of variables. Routh's equations Poisson's identity. Jacobi-Poisson Theorem. Brachistochrone problem. Configuration space and system point.

Special theory of relativity, Galilean transformation, Basic postulates of relativity, Lorentz transformation, Consequences of Lorentz transformation, Relativistic momentum: variation of mass with velocity, relativistic force, work and energy.

Variation of functional, Necessary and sufficient conditions for extrema, Euler-Lagrange's equations and its Applications: Geodesic, minimum surface of revolution, Brachistochrone problem and other boundary value problems in ordinary and partial differential equations.

References:

1. Goldstein, H., Classical Mechanics, Dover.
2. Arnold, V.I.(Vogtmann, K., Weinstein, A.), Mathematical Methods of Classical Mechanics, Springer(GTM), 1989.

Further Reading:

1. Rana, N.C. and Jog, P.S., Classical Mechanics, Tata McGraw Hill..
2. Louis, N.H.and Finch, J.D., Analytical Mechanics.
3. Ramsay, A.S., Dynamics, Part-II.

SEMESTER II

Duration: 6 Months (Including Examinations)
Total Marks: 400, Total No. of Lectures: 70 (70 Hours per paper)

	Papers	Topics	Marks (Credit)
Semester II	Core - 5	Complex Analysis	100 (5)
	Core - 6	Linear Algebra	100 (5)
	Core - 7	Partial Differential Equations	100 (5)
	Core - 8	Continuum Mechanics	100 (5)

Core - 5
COMPLEX ANALYSIS
(70 LECTURES)

Complex Numbers:
Complex Plane, Stereographic Projection.

Complex Differentiation :
Derivative of a complex function, Comparison between differentiability in the real and complex senses, Comparison between the real and complex differentiability via R-linear and C-linear maps, Cauchy-Riemann equations, Necessary and sufficient criterion for complex differentiability, Analytic functions, Entire functions, Harmonic functions and Harmonic conjugates.

Complex Functions and Conformality :
Polynomial functions, Rational functions, Power series, Exponential, Logarithmic, Trigonometric and Hyperbolic functions, Branch of a logarithm, Conformal maps, Möbius Transformations.

Complex Integration :
The complex integral (over piecewise C^1 curves), Cauchy's Theorem and Integral Formula, Power series representation of analytic functions. The difference between Real Analytic functions and C^∞-functions over R. Real Analyticity vs. Complex Analyticity. Morera's Theorem, Goursat's Theorem, Liouville's Theorem, Fundamental Theorem of Algebra, Zeros of analytic functions, Identity Theorem, Weierstrass Convergence Theorem, Maximum Modulus Principle and its applications, Schwarz's Lemma, Index of a closed curve, Contour, Index of a contour, Simply connected domains, Cauchy's Theorem for simply connected domains.

Singularities:

Definitions and Classification of singularities of complex functions, Isolated singularities, Uniform convergence of sequences and series. Laurent series, Casorati-Weierstrass Theorem, Poles, Residues, Residue Theorem and its applications to contour integrals, Meromorphic functions, Applications of Argument Principle, Applications of Rouche's Theorem.

References:

1. Conway, J.B., Functions of One Complex Variable, Second Edition, Narosa Publishing House, 1973.
2. Marsden, J.E. and Ho_man, M.J., Basic Complex Analysis, Third Edition, W. H. Freeman and Company, New York, 1999.

Further Reading:

1. Sarason, D., Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
2. Ahlfors, L.V., Complex Analysis, McGraw-Hill, 1979.
3. Rudin, W., Real and Complex Analysis, McGraw- Hill Book Co., 1966.
4. Hille, E., Analytic Function Theory (2 vols.), Gonn & Co., 1959.
5. Gamelin, T.W., Complex Analysis, Springer, 2001.
6. Bak, J. and Newman, D.J., Complex Analysis, Springer, 2010.
7. Ponnusamy, S., Foundations of Complex Analysis, Narosa, 2008.

Core - 6

LINEAR ALGEBRA

(70 LECTURES)

Review of Vector Spaces:

Vector spaces over a field, subspaces. Sum and direct sum of subspaces. Linear span. Linear dependence and independence. Basis. Finite dimensional spaces. Existence theorem for bases in the finite dimensional case. Invariance of the number of vectors in a basis, dimension. Existence of complementary subspace of any subspace of a finite dimensional vector space. Dimensions of sums of subspaces. Quotient space and its dimension.

Matrices and Linear Transformations:

Matrices and linear transformations, change of basis and similarity. Algebra of linear transformations. The rank-nullity theorem. Change of basis. Isomorphism Theorems. Dual space. Bi-dual space and natural isomorphism. Adjoint of linear transformations. Eigenvalues and eigenvectors of linear transformations. Determinants. Characteristic and minimal polynomials of linear transformations, Cayley-Hamilton Theorem. Annihilators. Diagonalization of operators. Invariant subspaces and decomposition of operators. Canonical forms.

Inner Product Spaces:

Inner product spaces. Cauchy-Schwartz inequality. Orthogonal vectors and orthogonal complements. Orthonormal sets and bases. Bessel's inequality. Gram-Schmidt orthogonalization method. Hermitian, Self-Adjoint, Unitary, and Orthogonal transformation for complex and real spaces. Bilinear and Quadratic forms, Real quadratic forms.

References:

1. Friedberg, S.H., Insel, A.J. and Spence, L.J., Linear Algebra, Prentice Hall of India, Fourth Edition, 2004.
2. Kumaresan, S., Linear Algebra, A Geometric Approach, Prentice Hall of India, Fourth Printing, 2003.

Further Reading:

1. Artin, M., Algebra, Prentice Hall of India, 2007.
2. Halmos, P.R., Finite Dimensional Vector Spaces, Springer, 2013.
3. Roman, S., Advanced Linear Algebra, Springer, 2007.
4. Curtis, C.W., Linear Algebra : An Introductory Approach, Springer (SIE), 2009.
5. Hoffman, K. and Kunze, R., Linear Algebra, Prentice Hall of India.
6. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.
7. Apostol, T.M., Calculus Vol. I & II, John Wiley and Sons, 2011.

Core -7

PARTIAL DIFFERENTIAL EQUATIONS

(70 LECTURES)

First Order PDE.:
Formation and solution of PDE, Integral surfaces, Cauchy Problem order equation, Orthogonal surfaces, First order non-linear PDE, Characteristics, Compatible system, Charpit's method. Classification and canonical forms of PDE.

Second Order Linear PDE:
Classification, reduction to normal form; Solution of equations with constant coefficients by (i) factorization of operators (ii) separation of variables.

Elliptic Differential Equations:
Derivation of Laplace and Poisson equation, Boundary Value Problem, Separation of Variables, Dirichlets Problem and Neumann Problem for a rectangle, Interior and Exterior Dirichlets problems for a circle, Interior Neumann problem for a circle, Solution of Laplace equation in Cylindrical and spherical coordinates,
Examples.

Parabolic Differential Equations:
Formation and solution of Diffusion equation, Dirac- Delta function, Separation of variables method, Solution of Diffusion Equation in Cylindrical and spherical coordinates, Examples.

Hyperbolic Differential Equations:
Formation and solution of one-dimensional wave equation, canonical reduction, Initial Value Problem, D'Alembert's solution, Vibrating string, Forced Vibration, Initial Value Problem and Boundary Value Problem for two- dimensional wave equation, Periodic solution of one-dimensional wave equation in cylindrical and spherical coordinate systems, vibration of circular membrane, Uniqueness of the solution for the wave equation, Duhamel's Principle, Examples.

Green's Function:
Green's function for Laplace Equation, methods of Images, Eigen function Method, Green's function for the wave and Diffusion equations. Laplace Transform method: Solution of Diffusion and Wave equation by Laplace Transform.

References:

1. Sneddon, I.N., Elements of Partial Differential Equations, McGraw Hill.

Further Reading:

1. Williams, W.E., Partial Differential Equations.
2. Miller, F.H., Partial Differential Equations.
3. Petrovsky, I.G., Lectures on Partial Differential Equations.
4. Courant & Hilbert, Methods of Mathematical Physics, Vol-II.
5. Rao, K.S., Introduction to Partial Differential Equations, Prentice Hall.
6. Amaranath, T., An elementary course in Partial Differential Equations, Narosa

Core - 8

CONTINUUM MECHANICS

(70 LECTURES)

Principles of continuum mechanics, axioms. Forces in a continuum. The idea of internal stress. Stress tensor. Equations of equilibrium. Symmetry of stress tensor. Stress transformation laws. Principal stresses and principal axes of stresses. Stress invariants. Stress quadric of Cauchy. Shearing stresses. Mohr's stress circles.

Deformation. Strain tensor. Finite strain components in rectangular Cartesion coordinates. Infinitesimal strain components. Geometrical interpretation of infinitesimal strain components. Principal strain and principal axes of strain. Strain invariants. The compatiability conditions. Compatibility of strain components in three dimensions.

Constitutive equations. Inviscid fluid. Circulation. Kelvins energy theorem. Constitutive equation for elastic material and viscous fluid. Navier and Stokes equations of motion.

Motion of deformable bodies. Lagrangian and Eulerian approaches to the study of motion of continua. Material derivative of a volume integral. Equation of continuity. Equations of motion. Equation of angular momentum. Equation of Energy. Strain energy density function.

References :

1. Fung, Y. C., A first course in continuum mechanics.
2. Eringen, A. C., Mechanics of continua.

Further Reading:

1. Sedov, L. I., A course in continuum mechanics. Vol – I.
2. Prager, W., Mechanics of continuous media.