

**SYLLABUS  
FOR  
M.SC. IN MATHEMATICS**

**(ON CHOICE BASED CREDIT SYSTEM)**

**A FOUR SEMESTERS COURSE**

*(Effective from the academic session 2020 – 2021 and onwards)*



**COOCH BEHAR PANCHANAN BARMA UNIVERSITY**  
**COOCH BEHAR, WEST BENGAL**

# **COOCH BEHAR PANCHANAN BARMA UNIVERSITY**

## **Syllabus for M.Sc. in Mathematics**

### **OBJECTIVE:**

The duration of the Post Graduate course in Mathematics of Cooch Behar Panchanan Barma University is two years with Semester-I, Semester-II, Semester-III and Semester-IV each of six months duration.

Syllabus for the P.G. course in Mathematics is hereby framed following the guidelines of UGC according to the following schemes and structures. All the students admitted to PG course in Mathematics shall take courses of Semester-I, Semester-II, Semester-III and Semester-IV.

### **SCHEME:**

Total Marks = 1600 with 400 marks in each semester comprising of four papers with 100 marks in each paper. 20% of the total marks is allotted for Continuous Evaluation and 5% is allotted for regular attendance. Students have to take four core papers in both the Semesters I and II, and one core paper with one Generic Elective paper in Semester-III and IV respectively. Also, in Semester –III and IV, they have to take two papers from Discipline Centric Electives. In the beginning of the Semester-III, the department will offer a cluster of Discipline Centric Electives and the students will have to choose two among them according to the norms to be decided by the Department in each year. The norms of the distribution of Discipline Centric Electives to the students will be decided by the Department depending upon the availability of teachers.

In Semester-IV, there will be a dissertation and a seminar presentation based on the Discipline Centric Electives carrying 25 marks each. The number of students for dissertation and seminar presentation will be more or less equally distributed to the faculty members. The topic of the dissertation and seminar presentation will be decided by the faculty members under whom the students will do their dissertation.

All the written papers will be evaluated by the internal examiners only. The continuous evaluations will be taken by the department and the answer scripts will be evaluated by the teachers of the department. It should be noted that some Discipline Centric Electives may be included in future as per the discretion of the department (subject to approval of the appropriate authority).

**SEMESTER COURSE STRUCTURE OF M.Sc. (MATHEMATICS)**

**SYLLABUS for SEMESTERS - I, II, III & IV**

<b>SEMESTERS</b>	<b>PAPERS</b>	<b>SUBJECTS</b>	<b>MARKS (CREDIT)</b>
<b>Semester I</b>	Core – 1	Real Analysis	100 (5)
	Core – 2	Abstract Algebra	100 (5)
	Core – 3	Ordinary Differential Equations and Special Functions	100 (5)
	Core – 4	Classical Mechanics	100 (5)
<b>Semester II</b>	Core – 5	Complex Analysis	100 (5)
	Core – 6	Linear Algebra	100 (5)
	Core – 7	Partial Differential Equations	100 (5)
	Core – 8	Continuum Mechanics	100 (5)
<b>Semester III</b>	Core – 9	Topology	100 (5)
	GE - 1	One have to choose from the pool of three subjects GE - 1(A), GE - 1(B) or GE - 1(C)	100 (5)
	DCE – 31(A,P)*	Discipline Centric Elective – 1 (I)	100 (5)
	DCE – 32 (A,P)*	Discipline Centric Elective – 2 (I)	100 (5)
<b>Semester IV</b>	Core – 10	Functional Analysis	100 (5)
	GE - 2	One have to choose from the pool of three subjects GE - 2(A), GE - 2(B) or GE - 2(C)	100 (5)
	DCE – 41(A,P)*	Discipline Centric Elective – 1 (II) with PG dissertation	100 (5)
	DCE – 42 (A,P)*	Discipline Centric Elective – 2 (II) with seminar presentation	100 (5)

**(A,P)\* : A - DCE from Applied Mathematics branches  
P – DCE from Pure Mathematics branches**

**PAPER WISE DISTRIBUTION OF MARKS**

<b>Sl. No.</b>	<b>Paper</b>	<b>Written</b>	<b>Continuous Evaluation</b>	<b>Attendance</b>	<b>Total</b>	<b>Credit</b>
<b>1.</b>	Core – 1	75	20	05	100	05
<b>2.</b>	Core – 2	75	20	05	100	05
<b>3.</b>	Core – 3	75	20	05	100	05
<b>4.</b>	Core – 4	75	20	05	100	05
<b>5.</b>	Core – 5	75	20	05	100	05
<b>6.</b>	Core – 6	75	20	05	100	05
<b>7.</b>	Core – 7	75	20	05	100	05
<b>8.</b>	Core – 8	75	20	05	100	05
<b>9.</b>	Core – 9	75	20	05	100	05
<b>10.</b>	GE - 1	75	20	05	100	05
<b>11.</b>	DCE – 31 (A,P)	75	20	05	100	05
<b>12.</b>	DCE – 32 (A,P)	75	20	05	100	05
<b>13.</b>	Core – 10	75	20	05	100	05
<b>14.</b>	GE - 2	75	20	05	100	05
<b>15.</b>	DCE – 41 (A,P)	Discipline Centric Elective – 1(II) : 50 (Written)  Dissertation: 25	20	05	100	05
<b>16</b>	DCE – 42 (A,P)	Discipline Centric Elective – 2(II) : 50 (Written)  Seminar presentation: 25	20	05	100	05
<b>Total</b>					<b>1600</b>	<b>80</b>

### **Discipline Centric Electives (DCE) FOR SEMESTERS III & IV:**

Two sets of Discipline Centric Electives have to be chosen from the following list either from **1 - 6** or from **7 - 12** at the beginning of Semester-III. The Discipline Centric Elective - 1(II) and Discipline Centric Elective - 2(II) in Semester - IV refer to the corresponding part of the Discipline Centric Elective - 1(I) and Discipline Centric Elective - 2(I) of Semester - III respectively. In Semester - IV, Discipline Centric Elective – 1 (II) contains dissertation of 25 marks. Each student has to submit a dissertation under the guidance of a faculty member of the department. Also, Discipline Centric Elective – 2(II) contains seminar presentation of 25 marks.

1. (I) PLASMA MECHANICS I (DCE – 31A)  
(II) PLASMA MECHANICS II (DCE – 41A)
2. (I) FLUID MECHANICS I (DCE – 32A)  
(II) FLUID MECHANICS II (DCE – 42A)
3. (I) NON-LINEAR DIFFERENTIAL EQUATIONS I  
(II) NON-LINEAR DIFFERENTIAL EQUATIONS II
4. (I) OPERATIONAL RESEARCH I  
(II) OPERATIONAL RESEARCH II
5. (I) QUANTUM MECHANICS I  
(II) QUANTUM MECHANICS II
6. (I) COMPUTATIONAL FLUID DYNAMICS I  
(II) COMPUTATIONAL FLUID DYNAMICS II
7. (I) ADVANCED COMPLEX ANALYSIS I (DCE – 31P)  
(II) ADVANCED COMPLEX ANALYSIS II (DCE – 41P)
8. (I) DIFFERENTIAL GEOMETRY I (DCE – 32P)  
(II) DIFFERENTIAL GEOMETRY II (DCE – 42P)
9. (I) ADVANCED REAL ANALYSIS I  
(II) ADVANCED REAL ANALYSIS II
10. (I) ADVANCED TOPOLOGY I  
(II) ADVANCED TOPOLOGY II
11. (I) OPERATOR THEORY I  
(II) OPERATOR THEORY II
12. (I) ADVANCED FUNCTIONAL ANALYSIS I  
(II) ADVANCED FUNCTIONAL ANALYSIS II

### GENERIC ELECTIVE FOR SEMESTER III :

In Semester – III, each student have to choose one Generic Elective (GE - 1) paper from the following three:

GE – 1 (A): Numerical Analysis with MATLAB Programming

GE – 1 (B): Mathematical Modeling

GE – 1 (C): Algebraic Coding Theory

### GENERIC ELECTIVE FOR SEMESTER IV :

In Semester – IV, each student have to choose one more Generic Elective (GE - 2) paper from the following three:

GE – 2 (A): Mathematical Methods

GE – 2 (B): Dynamical Systems

GE – 2 (C): Algebraic Aspects of Cryptology

### SEMESTER I

Duration: 6 Months (Including Examinations)  
Total Marks: 400, Total No. of Lectures: 70 (70 Hours) per paper

	<b>Papers</b>	<b>Topics</b>	<b>Marks (Credit)</b>
<b>Semester I</b>	Core – 1	Real Analysis	100 (5)
	Core – 2	Abstract Algebra	100 (5)
	Core – 3	Ordinary Differential Equations and Special Functions	100 (5)
	Core – 4	Classical Mechanics	100 (5)

**Core - 1**  
**REAL ANALYSIS**  
**(70 LECTURES)**

**Bounded Variation:**

Functions of Bounded Variation and their properties, Riemann- Stieltjes integrals and its properties, Absolutely Continuous Functions.

**The Lebesgue Measure:**

Lebesgue Measure: (Lebesgue ) Outer measure and measure on  $\mathbb{R}$ , Measurable sets form an  $\sigma$ -algebra, Borel  $\sigma$  - algebra, Borel sets, measure of open and closed sets, Existence of a non-measurable set, Measure space, Measurable Function and its properties, Borel measurable functions, Concept of Almost Everywhere (a.e.), sets of measure zero, Steinhaus Theorem, Sequence of measurable functions, Egorov's Theorem, Applications of Lusin Theorem.

**The Lebesgue Integral:**

Simple and Step Functions, Lebesgue integral of simple and step functions, Lebesgue integral of a bounded function over a set of finite measure, Bounded Convergence Theorem, Lebesgue integral of non-negative function, Fatou's Lemma, Monotone Convergence Theorem. The General Lebesgue integral: Lebesgue Integral of an arbitrary Measurable Function, Lebesgue Integrable functions. Dominated Convergence Theorem. Convergence in Measure. Riemann Integral as Lebesgue Integral. Product measure spaces, Fubini's Theorem (applications only).

**References:**

1. Apostol, T.M., Mathematical Analysis, Narosa Publishing House, 2002.
2. Royden, H.L., Fitzpatrick P.M., Real Analysis, 4th Edition, Pearson.
3. Aliprantis, C.D. & Burkinshaw, O., Principles of Real Analysis, Third Edition, Harcourt Asia Pvt. Ltd., 1998.
4. Jain, P. K. , Gupta, V. P. & Jain, P., Lebesgue measure and integration, Third Edition, New Age International.

**Further Reading:**

1. Halmos, P.R., Measure Theory, Springer, 2007.
2. Rudin, W., Principles of Mathematical Analysis, Tata McGraw Hill, 2001.
3. Rudin, W., Real and Complex Analysis, McGraw-Hill Book Co., 1966.
4. Tao, T., An Introduction to Measure Theory, American Mathematical Society.
5. Kolmogorov, A.N. & Fomin, S.V., Measures, Lebesgue Integrals, and Hilbert Space, Academic Press, New York & London, 1961.

6. Rana, I.K., An introduction to Measure and Integration, Second Edition, Narosa.
7. Barra, G.D., Measure Theory and Integration, Woodhead Pub.
8. Kingman, J.F.C. & Taylor, S.J., Introduction to Measure and Probability, Cambridge University Press, 1966.
9. Cohn, D.L., Measure Theory, Birkhauser, 2013.
10. Wheeden, R.L. & Zygmund, A., Measure and Integral, Monographs and Textbooks in Pure and Applied Mathematics, 1977.
11. Sohrab, H.H., Basic Real Analysis, Birkhauser, 2003.

## **Core - 2**

### **ABSTRACT ALGEBRA**

#### **(70 LECTURES)**

#### **Groups:**

Review of basic concepts of Group Theory: Lagrange's Theorem, Cyclic Groups, Permutation Groups and Groups of Symmetry:  $S_n$ ;  $A_n$ ;  $D_n$ , Conjugacy Classes, Index of a Subgroup, Divisible Abelian Groups. Homomorphism of Groups, Normal Subgroups, Quotient Groups, Isomorphism Theorems, Cayley's Theorem.

Direct Product and Semi-Direct Product of Groups, Fundamental Theorem (Structure Theorem) of Finite Abelian Groups, Cauchy's Theorem, Group Action, Sylow Theorems and their applications. Solvable Groups (Definition and Examples only). Field extension, Galois' theory. Modules.

#### **Rings:**

Ideals and Homomorphisms, Prime and Maximal Ideals, Quotient Field of an Integral Domain, Polynomial and Power Series Rings. Divisibility Theory : Euclidean Domain, Principal Ideal Domain, Unique Factorization Domain, Gauss' Theorem, Irrudicibility of polynomials, Chinese remainder theorem.

#### **References:**

1. Dummit, D.S., Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.
2. Malik, D.S., Mordesen, J.M. & Sen, M.K., Fundamentals of Abstract Algebra, The McGraw-Hill Companies, Inc, 1997.
3. Gallian, J., Contemporary Abstract Algebra, Narosa, 2011.
4. Herstein, I.N., Topics in Abstract Algebra, Wiley Eastern Limited.

#### **Further Reading:**

1. Roman, S., Fundamentals of Group Theory: An Advanced Approach, Birkhauser, 2012.
2. Sen M.K., Ghosh & S., Mukhopadhyay P., Topics in Abstract Algebra, Universities Press.
3. Rotman, J., The Theory of Groups: An Introduction, Allyn and Bacon, Inc., Boston.
4. Rotman, J., A First Course in Abstract Algebra, Prentice Hall, 2005.
5. Pinter, Charles. C., A Book of Abstract Algebra, McGraw Hill, 1982.

6. Fraleigh, J.B., A First Course in Abstract Algebra, Narosa.
7. Jacobson, N., Basic Algebra, I & II, Hindusthan Publishing Corporation, India.
8. Hungerford, T.W., Algebra, Springer.
9. Artin, M., Algebra, Prentice Hall of India, 2007.
10. Goldhaber, J.K. & Ehrlich, G., Algebra, The Macmillan Company, Collier-Macmillan Limited, London.
11. Gopalakrishnan, N.S., University Algebra, New Age International, 2005.

### **Core - 3**

## **ORDINARY DIFFERENTIAL EQUATIONS & SPECIAL FUNCTIONS**

**(70 LECTURES)**

#### **Ordinary Differential Equations:**

##### **Existence and Uniqueness:**

First order ODE, Initial value problems, Existence theorem, Uniqueness, basic theorems, Ascoli Arzela theorem (statement only), Theorem on convergence of solution of initial value problems. Picard – Lindelöf theorem (statement only), Peano's existence theorem (statement only) and corollaries.

##### **Boundary Value Problems for Second Order Equations:**

Ordinary Differential Equations of the Sturm-Liouville type and their properties - Application to Boundary Value Problems, Eigenvalues and Eigenfunctions, Orthogonality theorem, Expansion theorem. Green's function for Ordinary Differential Equations - Application to Boundary Value Problems.

#### **Special Functions:**

##### **Singularities:**

Fundamental System of Integrals, Singularity of a Linear Differential Equation. Solution in the neighbourhood of a singularity, Regular Integral, Equation of Fuchsian type, Series solution by Frobenius method.

##### **Legendre Polynomials:**

Legendre Functions, Generating Function, Legendre Functions of First & Second kind, Laplace Integral, Orthogonal Properties of Legendre Polynomials, Rodrigue's Formula.

##### **Bessel Functions:**

Bessel's Functions, Series Solution, Generating Function, Integral Representation of Bessel's Functions, Recurrence Relations, Asymptotic Expansion of Bessel Functions.

**Hermite Polynomial:**

Hermite equation and its solution, Generating function, Rodrigue's formula, Recurrence relations, Orthogonal Properties of Hermite Polynomials.

**Lagurre polynomial:**

Lagurre equation and its solution, Generating function, Recurrence relations, Orthogonal Properties of Hermite Polynomials.

**Hypergeometric Function:**

Hypergeometric Functions, Series Solution near zero, one and infinity. Integral Formula, Confluent Hypergeometric function, Integral representation of Hypergeometric function, Differentiation of Hypergeometric Function.

**References:**

1. Simmons, G.F., Differential Equations, Tata McGraw Hill.
2. Agarwal, Ravi P. & O' Regan D., An Introduction to Ordinary Differential Equations, Springer, 2000.
3. Churchill, R. V. & Brown, J. W., Fourier series and boundary value problems, Fifth Edition, McGraw Hill.
4. Codington, E.A & Levinson, N., Theory of Ordinary Differential Equation, McGraw Hill.

**Further Reading:**

1. Ince, E.L., Ordinary Differential Equation, Dover.
2. Ghatak, A. K., Goyal, I. C. & Chua, S. J., Mathematical Physics, Laxmi Publications
3. Estham, M.S.P., Theory of Ordinary Differential Equations, Van Nostrand Reinhold Compa.Ny, 1970.
4. Piaggio, H.T.H., An Elementary Treatise On Differential Equations And Their Applications, G. Bell And Sons, Ltd, 1949.
5. Hartman, P., Ordinary Differential Equations, SIAM, 2002.
6. Zill, D. G. & Cullen, M.R., Differential Equations with Boundary Value Problems, Brooks/Cole, 2009.

**Core - 4****CLASSICAL MECHANICS****(70 LECTURES)**

Dynamical systems, Generalized coordinates, Degrees of freedom, Principle of virtual work. D'Alembert's principle. Unilateral and bilateral constraints. Holonomic and non-holonomic system. Lagrange's equations for holonomic systems. Lagrange's equation for impulsive forces and for systems involving dissipative forces. Conservation theorems. Hamilton's principle and principle of least action. Hamilton's canonical equations. Canonical transformation with different generating functions. Lagrange and Poisson brackets and their properties. Hamilton-Jacobi

equations and separation of variables. Routh's equations, Poisson's identity. Jacobi-Poisson Theorem. Brachistochrone problem. Configuration space and system point.

Special theory of relativity, Galilean transformation, Basic postulates of relativity, Lorentz transformation, Consequences of Lorentz transformation, Relativistic momentum: variation of mass with velocity, relativistic force, work and energy.

Variation of functional, Necessary and sufficient conditions for extrema, Euler-Lagrange's equations and its Applications: Geodesic, minimum surface of revolution, Brachistochrone problem and other boundary value problems in ordinary and partial differential equations.

**References:**

1. Goldstein, H., Classical Mechanics, Dover.
2. Arnold, V.I., Mathematical Methods of Classical Mechanics, Springer (GTM), 1989.
3. Gupta, A. S., Calculus of variations with applications, Prentice Hall of India.

**Further Reading:**

1. Rana, N.C. & Jog, P.S., Classical Mechanics, Tata McGraw Hill.
2. Louis, N.H. & Finch, J.D., Analytical Mechanics, Cambridge University Press.
3. Ramsay, A.S., Dynamics, Part-II, CBS.

**SEMESTER II**

Duration: 6 Months (Including Examinations)

Total Marks: 400, Total No. of Lectures: 70 (70 Hours) per paper

	<b>Papers</b>	<b>Topics</b>	<b>Marks (Credit)</b>
<b>Semester II</b>	Core - 5	Complex Analysis	100 (5)
	Core - 6	Linear Algebra	100 (5)
	Core - 7	Partial Differential Equations	100 (5)
	Core - 8	Continuum Mechanics	100 (5)

**Core - 5**

**COMPLEX ANALYSIS**

**(70 LECTURES)**

**Complex Numbers:**

Complex Plane, Stereographic Projection.

**Complex Differentiation :**

Derivative of a complex function, Comparison between differentiability in the real and complex senses, Comparison between the real and complex differentiability via R-linear and C-linear maps, Cauchy-Riemann equations, Necessary and sufficient criterion for complex differentiability, Analytic functions, Entire functions, Harmonic functions and Harmonic conjugates.

**Complex Functions and Conformality :**

Polynomial functions, Rational functions, Power series, Exponential, Logarithmic, Trigonometric and Hyperbolic functions, Branch of a logarithm, Conformal maps, Möbius Transformations.

**Complex Integration :**

The complex integral (over piecewise  $C^1$  curves), Cauchy's Theorem and Integral Formula, Power series representation of analytic functions. The difference between Real Analytic functions and  $C^\infty$ -functions over  $\mathbb{R}$ . Real Analyticity vs. Complex Analyticity. Morera's Theorem, Goursat's Theorem, Liouville's Theorem, Fundamental Theorem of Algebra, Zeros of analytic functions, Identity Theorem, Weierstrass Convergence Theorem, Maximum Modulus Principle and its applications, Schwarz's Lemma, Index of a closed curve, Contour, Index of a contour, Simply connected domains, Cauchy's Theorem for simply connected domains.

**Singularities:**

Definitions and Classification of singularities of complex functions, Isolated singularities, Uniform convergence of sequences and series. Laurent series, Casorati-Weierstrass Theorem, Poles, Residues, Residue Theorem and its applications to contour integrals, Meromorphic functions, Applications of Argument Principle, Applications of Rouché's Theorem.

**References:**

1. Brown, J.W. & Churchill, R. V., Complex Variables and Applications, Eighth Edition, McGraw-Hill, 2009.
2. Ponnusamy, S., Foundations of Complex Analysis, Narosa, 2008.
3. Markusevich to be added.
4. Conway, J.B., Functions of One Complex Variable, Second Edition, Narosa Publishing House, 1973.
5. Marsden, J.E. & Hoffman, M.J., Basic Complex Analysis, Third Edition, W. H. Freeman and Company, New York, 1999.

**Further Reading:**

1. Ahlfors, L.V., Complex Analysis, McGraw-Hill, 1979.
2. Sarason, D., Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
3. Rudin, W., Real and Complex Analysis, McGraw-Hill Book Co., 1966.
4. Hille, E., Analytic Function Theory (2 vols.), Gonn & Co., 1959.
5. Gamelin, T.W., Complex Analysis, Springer, 2001.
6. Bak, J. & Newman, D.J., Complex Analysis, Springer, 2010.

**Core - 6**  
**LINEAR ALGEBRA**  
**(70 LECTURES)**

**Review of Vector Spaces:**

Vector spaces over a field, subspaces. Sum and direct sum of subspaces. Linear span. Linear dependence and independence. Basis. Finite dimensional spaces. Existence theorem for bases in the finite dimensional case. Invariance of the number of vectors in a basis, dimension. Existence of complementary subspace of any subspace of a finite dimensional vector space. Dimensions of sums of subspaces. Quotient space and its dimension.

**Matrices and Linear Transformations:**

Matrices and linear transformations, change of basis and similarity. Algebra of linear transformations. The rank-nullity theorem. Change of basis. Isomorphism Theorems. Dual space. Bi-dual space and natural isomorphism. Adjoint of linear transformations. Eigenvalues and eigenvectors of linear transformations. Determinants. Characteristic and minimal polynomials of linear transformations, Cayley-Hamilton Theorem. Annihilators. Diagonalization of operators. Invariant subspaces and decomposition of operators. Canonical forms.

**Inner Product Spaces:**

Inner product spaces. Cauchy-Schwartz inequality. Orthogonal vectors and orthogonal complements. Orthonormal sets and bases. Bessel's inequality. Gram-Schmidt orthogonalization method. Hermitian, Self-Adjoint, Unitary, and Orthogonal transformation for complex and real spaces. Bilinear and Quadratic forms, Real quadratic forms.

**References:**

1. Friedberg, S.H., Insel, A.J. & Spence, L.J., Linear Algebra, Prentice Hall of India, Fourth Edition, 2004.
2. Hoffman, K. & Kunze, R., Linear Algebra, Prentice Hall of India.
3. Kolman, B. & Hill, D., Elementary Linear Algebra, Pearson.
4. Kumaresan, S., Linear Algebra, A Geometric Approach, Prentice Hall of India, Fourth Printing, 2003.

**Further Reading:**

1. Halmos, P.R., Finite Dimensional Vector Spaces, Springer, 2013.
2. Roman, S., Advanced Linear Algebra, Springer, 2007.
3. Curtis, C.W., Linear Algebra : An Introductory Approach, Springer (SIE), 2009.

**Core -7**  
**PARTIAL DIFFERENTIAL EQUATIONS**  
**(70 LECTURES)**

**First Order PDE.:**

Classification and canonical forms of PDE.

**Second Order Linear PDE:**

Classification, reduction to normal form; Solution of equations with constant coefficients by (i) factorization of operators (ii) separation of variables.

**Elliptic Differential Equations:**

Derivation of Laplace and Poisson equation, Boundary Value Problem, Separation of Variables, Dirichlets Problem and Neumann Problem for a rectangle, Interior and Exterior Dirichlets problems for a circle, Interior Neumann problem for a circle, Solution of Laplace equation in Cylindrical and spherical coordinates,

Examples.

**Parabolic Differential Equations:**

Formation and solution of Diffusion equation, Dirac- Delta function, Separation of variables method, Solution of Diffusion Equation in Cylindrical and spherical coordinates, Examples.

**Hyperbolic Differential Equations:**

Formation and solution of one-dimensional wave equation, canonical reduction, Initial Value Problem, D'Alembert's solution, Vibrating string, Forced Vibration, Initial Value Problem and Boundary Value Problem for two-dimensional wave equation, Periodic solution of one-dimensional wave equation in cylindrical and spherical coordinate systems, vibration of circular membrane, Uniqueness of the solution for the wave equation, Duhamel's Principle, Examples.

**Green's Function:**

Green's function for Laplace Equation, methods of Images, Eigen function Method, Green's function for the wave and Diffusion equations. Laplace Transform method: Solution of Diffusion and Wave equation by Laplace Transform.

**References:**

1. Sneddon, I.N., Elements of Partial Differential Equations, McGraw Hill.
2. Rao, K.S., Introduction to Partial Differential Equations, Prentice Hall.
3. Prasad, P. & Rabindran, R., Partial Differential Equations, New Age International.

**Further Reading:**

1. Williams, W.E., Partial Differential Equations, Clarendon Press.
2. Miller, K. S., Partial Differential Equations in engineering problems, Dover.
3. Petrovsky, I.G., Lectures on Partial Differential Equations, Dover.
4. Courant & Hilbert, Methods of Mathematical Physics, Vol-II, Willey-VCH.
5. Amaranath, T., An elementary course in Partial Differential Equations, Narosa

**Core - 8****CONTINUUM MECHANICS****(70 LECTURES)**

Principles of continuum mechanics, axioms. Forces in a continuum. The idea of internal stress. Stress tensor. Equations of equilibrium. Symmetry of stress tensor. Stress transformation laws.

Principal stresses and principal axes of stresses. Stress invariants. Stress quadric of Cauchy. Shearing stresses. Mohr's stress circles.

Deformation. Strain tensor. Finite strain components in rectangular Cartesian coordinates. Infinitesimal strain components. Geometrical interpretation of infinitesimal strain components. Principal strain and principal axes of strain. Strain invariants. The compatibility conditions. Compatibility of strain components in three dimensions.

Constitutive equations. Inviscid fluid. Circulation. Kelvin's energy theorem. Constitutive equation for elastic material and viscous fluid. Navier-Stokes equations of motion.

Motion of deformable bodies. Lagrangian and Eulerian approaches to the study of motion of continua. Material derivative of a volume integral. Equation of continuity. Equations of motion. Equation of angular momentum. Equation of Energy. Strain energy density function.

**References :**

1. Fung, Y. C., A first course in continuum mechanics, Pearson.
2. Eringen, A. C., Mechanics of continua, John Wiley and Sons.
3. Ramsay, A.S., Dynamics, Part-II, Cambridge University Press.

**Further Reading:**

1. Sedov, L. I., Mechanics of continuous media, Vol – I, World Scientific.
2. Prager, W., Introduction to Mechanics of continua, Dover.
3. Mase, G. E., Theory and problems of continuum mechanics, McGraw Hill.

**SEMESTER III**

Duration: 6 Months (Including Examinations)  
 Total Marks: 400, Total No. of Lectures: 70 Hours per paper

	<b>Papers</b>	<b>Topics</b>	<b>Marks (Credit)</b>
<b>Semester III</b>	Core – 9	Topology	100 (5)
	GE – 1	GE – 1(A)/ GE – 1(B)/ GE – 1(C)	100 (5)
	DCE – 31(A,P)	Discipline Centric Elective – 1 (I)	100 (5)
	DCE – 32 (A,P)	Discipline Centric Elective – 2 (I)	100 (5)

**Core - 9**  
**TOPOLOGY**  
**(70 LECTURES)**

**Review of Metric Spaces:**

Examples and its properties.

**Topological Spaces and Continuous Functions:**

Topology on a set, Examples of Topologies (Topological Spaces): Discrete Topology, Indiscrete Topology, Finite Complement Topology, Countable Complement Topology, Topologies on the Real Line :  $\mathbb{R}_l$ ,  $\mathbb{R}_K$ ,  $\mathbb{R}$  with usual Topology etc., Finer and Coarser Topologies, Basis and Sub basis for a topology. Product topology on  $X \times Y$ , Subspace Topology.

Interior Points, Limit Points, Derived Set, Boundary of a set, Closed Sets, Closure and Interior of a set, Kuratowski closure operator and the generated topology.

Continuous Functions, Rules for Constructing Continuous Functions: Inclusion Map, Composition, by restricting the domain, by restricting/expanding the range, Pasting Lemma, Open maps, Closed maps and Homeomorphisms, Embedding of a Topological Space into another Topological Space (examples only).

(Infinite) Product Topology : Sub basis for product Topology defined by Projection Maps, Box Topology, Metric Topology.

**Connectedness and Compactness:**

Connected and Path Connected Spaces: Definitions, Examples and its simple properties, Connected subsets of the real line, Introduction to Components and Path Components, Local Connectedness. Compact Spaces, Compact subsets of the real line, Heine-Borel Theorem, Separation Axioms.

**References :**

1. Munkres, J.R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
3. Kumaresan, S., Topology of Metric Spaces, Narosa Publishing House, 2010.
4. Kelley, J.L., General Topology, Dover.

**Further Reading:**

1. Dugundji, J., Topology, Allyn and Bacon, 1966.
2. Young, J.G., Topology, Addison-Wesley Reading, 1961.
3. Willard, S., General Topology, Dover.
4. Engelking, R., General Topology, Polish Scientific Pub.
5. Sierpinski, W., Introduction to General Topology, The University of Toronto Press, Canada.
6. Kuratowski, K., General Topology, Vol. I, Academic Press, New York and London.

## GE – 1 (A)

### NUMERICAL ANALYSIS with MATLAB PROGRAMMING

[45 (Theory) + 30 (Practical) = 75 MARKS, 70 LECTURES]

#### Numerical Analysis (45 Marks, 40 Lectures)

##### **Numerical Solution of System of Linear Equations:**

Triangular factorization methods, Matrix inversion method, Iterative methods- Jacobi method, Gauss Jacobi method, Gauss-Seidel method, Successive over relaxation (SOR) method and convergence condition of Iterative methods, Rate of convergence of methods.

##### **Solution of Non-linear Equations:**

Methods of Iteration: Tchebyshev method, Multipoint method, Modified Newton-Raphson method (for real roots simple or repeated), Rate of convergence of all these methods.

##### **System of Non-linear Equations:**

Newton's Method, Quasi-Newton's method.

##### **Numerical Solution of Initial Value Problem for ODE:**

First order Equation: Multistep predictor-corrector methods, Convergence and stability.

##### **Two Point Boundary Value Problem for ODE:**

Finite difference method, Shooting Method.

##### **Numerical Solution of PDE by Finite Difference Method:**

Parabolic equation in one dimension (Heat equation), Explicit finite difference method, Implicit Crank-Nickolson method, One dimensional Wave equation - Finite difference method, Convergence and Stability.

#### MATLAB programming (30 Marks, 30 Lectures)

MATLAB preliminaries, Creating arrays and their manipulations, MATLAB scripts and functions, Plotting in two and three dimensions, solving ordinary and partial differential equations (IVPs and BVPs), solving system of ordinary differential equations.

##### **References:**

1. Jain, M.K., Iyenger, S.R.K. & Jain, R.K., Numerical Methods for Scientific and Engineering Computation, New Age International.
2. Higham, D.J. & Higham, N.J., MATLAB guide, Vol. 150, SIAM, 2016.

##### **Further Reading:**

1. Atkinson, K.E., An Introduction to Numerical Analysis, John Wiley & Sons, 1989.
2. Smith, G.D., Numerical Solution of Partial Differential Equations, Clarendon Press.
3. Berzin & Zhidnov, Computing methods, Pergamon.
4. Isacson & Keller, Analysis of Numerical methods, Dover.

5. Ralston & Rabinowitz, A First Course in Numerical Analysis, Dover.
6. Jain, M.K., Numerical Solution of Differential Equations, New Age International.
7. Fox, L., Numerical Solution of Ordinary and Partial Differential Equations, Oxford Univ. Press.
8. Moore, H., MATLAB for Engineers, Pearson, 2017.
9. Cooper, J.M., Introduction to partial differential equations with MATLAB, Springer Science & Business Media, 2012.
10. Hahn, B. & Valentine, D., Essential MATLAB for engineers and scientists, Academic Press, 2016.

## **GE - 1 (B)**

### **MATHEMATICAL MODELING**

**(70 LECTURES)**

#### **Unit-I**

Simple situations requiring mathematical modelling, techniques of mathematical modeling, Classifications, Characteristics and limitations of mathematical models, Some simple illustrations. Mathematical modelling in population dynamics, Mathematical modelling of epidemics through systems of ordinary differential equations of first order Mathematical Models in Medicine, Battles and international Trade in terms of Systems of ordinary differential equations.

#### **Unit-II**

The need for Mathematical modelling through difference equations, linear growth and decay models, Nonlinear growth and decay models, Basic theory of linear difference equations with constant coefficients, Mathematical modelling through difference equations in economics and finance.

#### **Unit-III**

Mathematical modelling through difference equations in population dynamics and genetics. Mathematical Modelling through difference equations in probability theory. Miscellaneous examples of Mathematical modelling through difference equations.

#### **Unit-IV**

Situations that can be modelled through graphs, Mathematical models in terms of directed graphs Mathematical models in terms of signed graphs, Mathematical models in terms of weighted graphs.

#### **References:**

1. Kapur J. N., Mathematical Modelling, New Age International, 1988.
2. Rutherford, A., Mathematical Modelling Techniques. Courier Corporation, 2012.
3. Bender, E. A., An Introduction to Mathematical Modelling. Courier Corporation, 2000.

### **Further Reading:**

1. Clive, L. D., Principles of Mathematical Modelling. Elsevier, 2004.
2. Meerschaert, M. M., Mathematical Modelling. Academic Press, 2013.

**GE – 1(C)**

### **ALGEBRAIC CODING THEORY**

**(70 LECTURES)**

The Communication channel. The Coding Problem. Types of Codes. Block Codes. Error-Detecting and Error-Correcting Codes. Linear Codes. The Hamming Metric. Description of Linear Block Codes by Matrices. Dual Codes. Standard Array. Syndrome. Step-by-step Decoding Modular Representation. Error-Correction Capabilities of Linear Codes. Bounds on Minimum Distance for Block Codes. Plotkin Bound. Hamming Sphere packing Bound. Varshamov-Gilbert-Sacks Bound. Bounds for Burst-Error Detecting and Correcting Codes. Important Linear Block Codes. Hamming Codes. Golay Codes. Perfect Codes. Quasi—perfect Codes. Reed-Muller Codes. Codes derived from Hadamard Matrices. Product Codes Concatenated Codes. Tree Codes. Convolutional Codes. Description of Linear Tree and Convolutional Codes by Matrices. Standard Array. Bounds on Minimum distance for Convolutional Codes. V.G.S., bound. Bounds for Burst-Error Detecting and Correcting Convolutional Codes. The Lee metric, packing bound for Hamming code w.r.t. Lee metric.

The Algebra of polynomial residue classes. Galois Fields. Multiplicative group of a Galois field. Cyclic Codes. Cyclic Codes as Ideals. Matrix Description of Cyclic Codes. Hamming and Golay Codes as Cyclic Codes. Error Detection with cyclic Codes. Error-Correction procedure for Short Cyclic Codes. Shortened Cyclic Codes. Pseudo Cyclic Codes. Code symmetry. Invariance of Codes under transitive group of permutations. **Bose-Chaudhuri-Hocquenghem** (BCH) Codes. Majority-Logic Decoding. BCH bounds. Reed-Solomon (RS) Codes. Majority-Logic Decodable Codes. Majority-Logic Decoding. Singleton bound. The Griesmer bound. Maximum-distance Separable (MDS) Codes. Generator and Parity-check matrices of MDS Code. Weight Distribution of MDS Code. Necessary and Sufficient conditions for linear code to be an MDS Code. MDS codes from RS codes. Abramson Codes. Close-loop burst-Error correcting codes (Fire codes). Error Locating Codes.

### **References:**

1. Roman, S., Coding and Information Theory, Springer-Verlag.
2. Hamming, R., Coding and Information Theory, Prentice Hall.
3. MacWilliams, F. J. & Sloane, N.J.A., The Theory of Error-Correcting Codes, North- Holland Mathematical Library.

4

**Further Reading:**

1. Biggs, N. L., Codes- An Introduction to Information Communication and Cryptography:  
Springer Undergraduate Mathematics Series

**DCE - 1(I)**  
**(70 LECTURES)**

**&**

**DCE - 2(I)**  
**(70 LECTURES)**

**SEMESTER IV**

Duration: 6 Months (Including Examinations)  
Total Marks: 400, Total No. of Lectures: 70 Hours per paper

	<b>Papers</b>	<b>Topics</b>	<b>Marks (Credit)</b>
<b>Semester IV</b>	Core – 10	Functional Analysis	100 (5)
	GE – 2	GE – 2(A)/ GE – 2(B)/ GE – 2(C)	100 (5)
	DCE – 41(A,P)*	Discipline Centric Elective – 1 (II) with PG dissertation	100 (5)
	DCE – 42 (A,P)*	Discipline Centric Elective – 2 (II) with seminar presentation	100 (5)

**Core - 10****FUNCTIONAL ANALYSIS**  
**(70 LECTURES)****Banach Spaces:**

Normed Linear Spaces and its properties, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Riesz Lemma. Bounded Linear Transformations. Uniform Boundedness Theorem, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, Necessary and sufficient conditions for Bounded (Continuous) and Unbounded (Discontinuous) Linear functionals in terms of their kernel. Hyperplane, Necessary and sufficient conditions for a subspace to be hyperplane. Applications of Hahn-Banach Theorem, Dual Space, Examples of Reflexive Banach Spaces.  $L^p$ -Spaces and their properties.

### **Hilbert Spaces:**

Real Inner Product Spaces and its Complexification, Cauchy-Schwarz Inequality, Parallelogram law, Pythagorean Theorem, Bessel's Inequality, Gram-Schmidt Orthogonalization Process, Hilbert Spaces, Orthonormal Sets, Complete Orthonormal Sets and Parseval's Identity, Orthogonal Complement and Projections. Riesz Representation Theorem for Hilbert Spaces, Adjoint of an Operator on a Hilbert Space with examples, Reflexivity of Hilbert Spaces, Definitions and examples of Self-adjoint Operators, Positive Operators, Projection Operators, Normal Operators and Unitary Operators. Introduction to Spectral Properties of Bounded Linear Operators.

### **References :**

1. Kreyszig, E., Introductory Functional Analysis and its Applications, John Wiley and Sons, New York, 1978.
2. Limaye, B.V., Functional Analysis, Wiley Eastern Ltd, 1981.
3. Lahiri, B. K., Elements of Functional Analysis, World Press.

### **Further Reading:**

1. Brown, A. & Percy, C., Introduction to Operator Theory I : Elements of Functional Analysis, Springer-Verlag New York, 1977.
2. Suhubi, E.S., Functional Analysis, Springer, New Delhi, 2009.
3. Aliprantis, C.D. & Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd., 1998.
4. Ponnusamy, S., Foundations of Functional Analysis, Narosa, 2011.
5. Goffman, C. & Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.
6. Bachman, G. & Narici, L., Functional Analysis, Academic Press, 1966.
7. Taylor, A.E., Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.
8. Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
9. Conway, J.B., A Course in Functional Analysis, Springer Verlag, New York, 1990.
10. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.

**GE – 2(A)**

**MATHEMATICAL METHODS**

**(70 LECTURES)**

### **Laplace Transform:**

Laplace transform: inverse transform (Bromwich formula), Convolution theorem, Application to ordinary and partial differential equations.

### **Fourier Transform:**

Fourier transform, Fourier Sine transform, Fourier Cosine Transform and their inversion formula, Convolution, Parseval's relation, Problems, Multiple Fourier transform (Definition), Application of these transforms to Heat, Wave and Laplace equations.

**Hankel Transform:**

Hankel transform, Inversion formula, Parseval's relation, Finite Hankel transform, Application to differential equations.

**Integral Equation:**

Basic concepts, Classification of Integral equations.

Volterra integral equations: Conversion of IVPs to Volterra integral equations, Resolvent kernel, Method of successive approximations, Singular integral equations, Convolution type equations, Volterra equation of first kind, Abel's integral equation.

Fredholm integral equations: Fredholm equations of first and second kind, Conversion of BVPs to Fredholm integral equations, the method of Fredholm determinants, Iterated kernels, Integral equations with degenerate kernels, Eigen values and eigen functions of a Fredholm integral equations, Fredholm alternative, Construction of Green's function for Boundary Value Problem.

**References:**

1. Davies, B., Integral Transforms and Their Applications, Springer.
2. Debnath, L. & Bhatta, D., Integral Transforms and Their Applications, CRC Press.
3. Lovitt, W.V., Linear Integral Equations, Dover.
4. Tricomi, F.G., Integral Equations, Dover.
5. Andrews, L. and Shivamoggi, V.K., Integral Transforms for Engineers, SPIE Press.
6. Mikhlin, S. G., Integral equations and their applications to certain problems in mechanics, Mathematical Physics and Technology, Cambridge University Press.

**Further Reading:**

1. Sneddon, I.N., The Uses of Integral Transforms, McGraw Hill.
2. Tranter, C.J., Integral Transforms in Mathematical Physics, John Wiley & sons..
3. Sneddon, I.N., Fourier Transform, Dover.
4. Pinkus, A. & Zafrany, S., Fourier Series and integral transforms, Cambridge University Press.
5. Courant, R. & Hilbert, D., Methods of Mathematical Physics, Vol - I & II, TBS.
6. Harper, C., Introduction to Mathematical Physics, Prentice Hall of India.

**GE – 2(B)****DYNAMICAL SYSTEMS****(70 LECTURES)****Linear systems:**

Linear autonomous systems, existence, uniqueness and continuity of solutions, diagonalization of linear systems, fundamental theorem of linear systems, the phase paths of linear autonomous plane systems, complex eigen values, multiple eigen values, similarity of matrices and Jordan

canonical form, stability theorem, reduction of higher order ODE systems to first order ODE systems, linear systems with periodic coefficients.

**Nonlinear systems:**

The flow defined by a differential equation, linearization of dynamical systems (two, three and higher dimension), Stability: (i) asymptotic stability (Hartman's theorem), (ii) global stability (Liapunov's second method).

**Periodic Solutions (Plane autonomous systems):**

Translation property, limit set, attractors, periodic orbits, limit cycles and separatrix, Bendixon criterion, Dulac criterion, Poincare-Bendixon Theorem, index of a point, index at infinity.

**Bifurcation and Center Manifolds:**

Stability and bifurcation, saddle-node, transcritical and pitchfork bifurcations, hopf bifurcation, center manifold (linear approximation).

**Nonlinear difference equations (Map):**

Steady states and their stability, the logistic difference equation, systems of nonlinear difference equations, stability criteria for second order equations, stability criteria for higher order system.

**Chaos:**

One-dimensional logistic map and chaos.

**References:**

1. Jordan, D. W. & Smith, P., Nonlinear Ordinary Equations- An Introduction to Dynamical Systems, Oxford University Press.
2. Perko, L., Differential Equations and Dynamical Systems, Springer-Verlag.

**Further Reading:**

1. Verhulst, F. , Nonlinear Differential Equations and Dynamical Systems, Springer-Verlag.
2. Yorke, A. S., Chaos - An Introduction to Dynamical Systems, Springer-Verlag.
3. Kelley, W. G. & Peterson, A. C., Difference Equations- An Introduction with Applications, Academic Press.

GE - 2(C)

**ALGEBRAIC ASPECTS OF CRYPTOLOGY**

**(70 LECTURES)**

**Theory (50 Marks)**

**Probability Theory :** Basic laws, Bernoulli and binomial random variables, the geometric distribution, Markov's inequality, Chebyshev's inequality, Chernoff's bound.

**Basic Algorithmic Number Theory :** Faster integer multiplication, extended Euclid's algorithm, quadratic residues, Legendre symbols, Jacobi symbols, Chinese Remainder theorem, fast modular exponentiation, choosing a random group element, finding a generator of a cyclic group, finding square roots modulo a prime  $p$ , polynomial arithmetic, arithmetic in finite fields, factoring polynomials over finite fields, isomorphisms between finite fields, computing order of an element, computing primitive roots, fast evaluation of polynomials at multiple points, primality testing, Miller-Rabin Test, Generating random primes, primality certificates, algorithms for factorizing, algorithm for computing discrete logarithms. Complexity analysis of various number theoretic algorithms.

**Public Key Cryptography and allied applications :** DLP, Diffie-Hellman key exchange, RSA, ElGamal, Rabin. Public key based signature schemes, Oblivious transfer protocols.

**Complexity Theory :** P, NP, P vs NP question, polynomial time reductions (emphasis on oracle machines), NP-Complete problems, randomized algorithms, probabilistic polynomial time, non-uniform polynomial time.

**Algebraic Geometry :** Affine Algebraic Sets, parametrizations of affine varieties, ordering of the monomials in  $K[X_1, X_2, \dots, X_n]$ , a division algorithm in  $K[X_1, X_2, \dots, X_n]$ , Monomial ideals and Dickson's Lemma, Hilbert Basis Theorem, Gröbner basis, properties, Buchberger's Algorithm.

**Practical (25 marks)**

C implementation of various primitives for cryptographic schemes.

**References:**

1. Galbraith, S. D., Mathematics of Public Key Cryptography, Cambridge University Press.
2. Stinson, D. R., Cryptography - Theory & Practice, CRC Press.
3. Hoffstein, J., Pipher, J. & Silverman, J. H., An Introduction to Mathematical Cryptography, Springer.

4. Katz, J. & Lindell, Y., Introduction to Modern Cryptography, Chapman & Hall/CRC.
5. Koblitz, N., A course in number theory and cryptography, Springer-Verlag.

**Further Reading:**

1. Burton, D. M., Elementary Number Theory, Wm. C. Brown Publishers, Dulreque, Iowa, 1989.
2. Kenneth, H. R., Elementary Number Theory & Its Applications, AT&T Bell Laboratories, AdditionWesley Publishing Company.
3. Ireland, K. & Rosen, M., A Classical Introduction to Modern Number Theory, Springer-Verlag.
4. Mollin, R. A., Advanced Number Theory with Applications; CRC /Chapman & Hall.
5. Alaca, S. & Williams, K. S., Introduction to Algebraic Number Theory, Cambridge University Press.
6. Goldman, J. R., The Queen of Mathematics : a historically motivated guide to number theory; A K Peters Ltd.

**DCE - 1(II) with dissertation**  
**[50 (Theory) + 25( dissertation)**  
**= 75 MARKS, 70 LECTURES]**

&

**DCE - 2(II) with seminar presentation**  
**[50 (Theory) + 25 (Seminar presentation)**  
**= 75 MARKS, 70 LECTURES]**

**DISCIPLINE CENTRIC ELECTIVES FOR SEMESTERS III & IV**

**Applied Mathematics branches (A):**

**PLASMA MECHANICS I**

Definition of Plasma as an ionized gas, Saha's equation of ionization, Occurrence of plasma in nature, Plasma as mixture of different species of charged particles.

Applications: Space Plasma, Solar Plasma, Gravitational Plasma, Laboratory Plasma,

Concept of temperature and density of plasma, Quasi-neutrality in plasma, The equilibrium state: Maxwellian Distribution, Debye Shielding, The plasma parameter and the criteria for plasma formation.

Orbit theory of plasma, Grad-B drift, Curvature drift, Polarization drift, Uniform  $\mathbf{E}$  and  $\mathbf{B}$  fields, Larmor orbits and guiding centers, The magnetic moment and the magnetization current, Non-uniform  $\mathbf{B}$  field, Non-uniform  $\mathbf{E}$  field, Time varying  $\mathbf{E}$  field, Time varying  $\mathbf{B}$  field, Adiabatic invariants,

Fluid description of Plasma: equation of continuity, equation of motion, equation of energy, fluid Drifts.

Plasma-Single fluid approach (MHD): Approximation for MHD, Basic equations for MHD:, Conservation of mass, Conservation of momentum, Conservation of energy, Conservation of magnetic flux, reduced MHD equation, generalization of Bernoulli's theorem, analogy of magnetic Reynolds number with hydrodynamics, Hartmann flow, Couette flow, Frozen-in-effect, Alfvén theorem, Ferraro's law of isorotation.

#### References:

1. Chen, F. F., Plasma Physics and controlled Fusion, Plenum Press, New York and London.
2. Bittencourt, J. A., Fundamental of Plasma Physics, Pergamon Press, New York and London.
3. Stix, T. H., Theory of Plasma waves, McGraw Hill.

## PLASMA MECHANICS II

Waves in Plasmas: Representation of waves, Group Velocity, Phase velocity, plasma oscillations, electron plasma waves, sound waves, Electron waves (electrostatic), Upper hybrid Oscillations, Ion waves (electrostatic), Ion-acoustic waves, Ion-cyclotron waves, Lower hybrid Oscillations, Electron waves (electromagnetic) : Light waves, O waves, X waves, R waves (whistler mode), L Waves, Ion waves (electromagnetic) : Alfvén wave, Magnetosonic wave

Elements of kinetic theory (Statistical approach): Single particle phase space, Volume elements, Distribution function, Characterization of plasma, Derivation of Boltzmann equation, Average values and Macroscopic variables, Derivation of Macroscopic equations (Moment equations):, Assumption on the nature of the distribution function to form a closed and consistent system of macroscopic equations (Equation of State), Cold Plasma limit, Vlasov – Boltzmann self-consistent equations in collision less plasma, Plasma Oscillations, Landau Damping.

Nonlinear wave processes in plasma: Derivation of KdV equation, KdV-Burger equation for ion-acoustic wave and their soliton solution.

The theta Pinch, The Z – Pinch, Pinched instability, Bennet's relation, Some MHD physics, Generalized Ohm's law, MHD equilibrium, Linear Stability, The Rayleigh – Taylor instability.

**References:**

1. Chen, F. F., Plasma Physics and controlled Fusion, Plenum Press, New York and London.
2. Bittencourt, J. A., Fundamental of Plasma Physics, Pergamon Press, New York and London.
3. Stix, T. H., Theory of Plasma waves, McGraw Hill.

**FLUID MECHANICS I**

**Kinematics of Fluids in motion:**

Lagrange's and Euler's methods in fluid motion. Equation of continuity and equation of motion, Boundary conditions, stream lines, path line and stream tube. Irrotational and rotational flows, Vorticity vector.

**Inviscid incompressible fluid flow:**

Euler's equation of motion, Bernoulli's pressure equation. Application of pressure equation, Euler's momentum theorem and D'Alembert's paradox. Theory of irrotational motion and circulation. Permanence irrotational motion.

**Motion in two-dimension:**

Irrotational motion. Stream function, velocity potential and Complex potential. Source, sink, doublet and their images with regards to a plane/circle. Permanence of irrotational motion, Circulation, Kelvin's circulation theorem, Cyclic constant and acyclic and cyclic motion. Kelvin's minimum energy theorem. Blasius theorem and its application. Milne-Thomson circle theorem. General motion of a cylinder in two dimensions. Motion of a cylinder in a uniform stream, Liquid streaming past a fixed circular cylinder and two coaxial cylinders.

**Vortex Motion:**

Vortex line, Vortex tube, Properties of the vortex, Strength of the vortex, Rectilinear vortices, Velocity component, centre of vortices. A case of two vortex filaments, vortex pair. Vortex doublet. Image of vortex filament with respect to a plane. An infinite single row of parallel rectilinear vortices of same strength. Two infinite row of parallel rectilinear vortices, Karman's vortex street. Rectilinear vortex with circular section. Rankine's combine vortex. Rectilinear vortices with elliptic section.

**References:**

1. Prandtl, L., Essential of fluid dynamics, Springer, 2004.
2. White, F.M., Viscous Fluid Flow, McGraw Hill, 1991.
3. Panton, R.L., Incompressible Flow, John Wiley and Sons, 1984.
4. Rosenhead, L., Laminar Boundary Layer, Dover, 1988.
5. Sherman, F.S., Viscous Flow (McGraw Hill).
6. Pai, S.I., Viscous Flow Theory, D.Van Nostrand, 1997.

7. Schlichting, H., Boundary Layer Theory, Springer, 2001.
8. Chorlton, F., Text Book of Fluid Dynamics, CBS Publ.
9. Love, A.E., A treatise on mathematical theory of elasticity, McGraw Hill Book Co., 1956.
10. Kondepudi, D. and Prigogine, I., Modern thermodynamics, John Wiley and Sons, Inc., 1998.
11. Landau, L.M. and Lifshitz, E.M., Fluid Mechanics, Butterworth Heinemann, 2005.
12. Ramsay, A.S., Dynamics, Part-II, Cambridge University Press.

## FLUID MECHANICS II

### Viscous Flow:

Navier-Stokes equations, Vorticity and circulation in viscous fluids. Reynolds number, Boundary conditions. Helmholtz equation for diffusion of vorticity, Dissipation of energy. Non-dimensional form of N.S. equations. Simple exact solutions of N-S equations: Parallel flow, Generalized Couette flow. Plane Poiseuille flow and simple Couette flow. Hagen Poiseuille flow through a circular pipe. Flow between parallel plates. Flow through pipes of circular, elliptic section under constant pressure gradient. Laminar flow between concentric rotating cylinder. Steady motion of a viscous fluid due to a slowly rotating sphere. Unsteady motion of a flat plate.

### Boundary Layer Theory:

Prandtl's concept of boundary layer. Two-dimensional boundary layer equation for flow over a plane wall, Boundary layer flow along a flat plate. Blasius-Topfer solution, 'similar solution' and separation of the boundary layer, boundary layer flow along the wall of a convergent channel, boundary layer flow past a wedge, Momentum and energy integral equation for the boundary layer. Von Karman Pohlhausen method.

### References:

1. Prandtl, L., Essential of fluid dynamics, Springer, 2004.
2. White, F.M., Viscous Fluid Flow, McGraw Hill, 1991.
3. Panton, R.L., Incompressible Flow, John Wiley and Sons, 1984.
4. Rosenhead, L., Laminar Boundary Layer, Dover, 1988.
5. Sherman, F.S., Viscous Flow (McGraw Hill).
6. Pai, S.I., Viscous Flow Theory, D. Van Nostrand, 1997.
7. Schlichting, H., Boundary Layer Theory, Springer, 2001.
8. Chorlton, F., Text Book of Fluid Dynamics, CBS Publ.
9. Love, A.E., A treatise on mathematical theory of elasticity, McGraw Hill Book Co., 1956.
10. Kondepudi, D. and Prigogine, I., Modern thermodynamics, John Wiley and Sons, Inc., 1998.
11. Landau, L.M. and Lifshitz, E.M., Fluid Mechanics, Butterworth Heinemann, 2005.
12. Ramsay, A.S., Dynamics, Part-II, Cambridge University Press.

## NON-LINEAR DIFFERENTIAL EQUATIONS I

Autonomous systems, Flows, Phase space, existence and uniqueness of solutions, stability, Lyapunov function, fixed points, saddle, nodes, focus, stable, unstable and centre subspaces, Hartmann-Grabmann Theorem (statement only), Examples, Linearisation, geometrical properties, averaging methods, perturbation method, method of multiscales, forced oscillations.

Poincare maps, periodic orbits, invariant sets, limit sets, attracting and repelling sets, Poincare-Bendixon theorem (statement only), bifurcations, simple examples, Hopf bifurcation.

### References:

1. Jordan, D. W. & Smith, P., Nonlinear Ordinary Differential Equations, OUP.
2. Coddington, E. A. & Levinson, N., Theory of Ordinary Differential Equations, McGraw Hill.
3. Devaney, R. L., An Introduction to Chaotic Dynamical Systems, Westview Press, 2003.

## NON-LINEAR DIFFERENTIAL EQUATIONS II

Fixed points, periodic points, orbits, stable and unstable sets, Logistic and other non-invertible maps, circle map, centre sets, symbolic dynamics, topological conjugacy, structural stability, Chaos, period doubling cascades, pitchfork, saddle node, transcritical bifurcations, bifurcations in ODE, Poincare sequence, Homoclinic paths, Horseshoe map, total auto-morphisms, chaos in nonlinear ODE.

### References:

1. Devaney, R. L., An Introduction to Chaotic Dynamical Systems, Westview Press, 2003.
2. Hasselblatt, B. & Katok, A., A first Course in Dynamics, CUP, 2010.
3. Holmgren, R. A., A first course in discrete dynamical systems, Springer.
4. Alligood, Sauer & York, Chaos, An introduction to dynamical systems, Springer.
5. Falconer, K., Foundation to fractal geometry, CUP.

## OPERATIONS RESEARCH I

### Dynamic Programming:

Introduction, Nature of dynamic programming, Deterministic processes, Non-Sequential discrete optimization, Allocation problems, Assortment problems, Sequential discrete optimization, Long-term planning problem, Multi-stage decision process, Application of Dynamic Programming in production scheduling and routing problems.

### Sequencing:

Problems with  $n$  jobs two machines,  $n$ -jobs three machines and  $n$ -jobs,  $m$ -machines.

### Inventory control:

Inventory control -Deterministic including price breaks and Multi-item with constraints, - Probabilistic (with and without lead time), Fuzzy and Dynamic inventory models.

**Network:**

PERT and CPM: Introduction, Basic difference between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM Network components and precedence relationships, Critical path analysis, Probability in PERT analysis.

**Replacement and Maintenance Models:**

Introduction, Failure Mechanism of items, Replacement of items deteriorates with time, Replacement policy for equipments when value of money changes with constant rate during the period, Replacement of items that fail completely individual replacement policy and group replacement policy, other replacement problems staffing problem, equipment renewal problem.

**References:**

1. Ronald, V. Hartley, Operations Research A Managerial Emphasis Goodyear Publishing Company Inc., 1976, California.
2. Beveridge & Schechter, Optimization Theory and Practice, McGraw Hill Kogakusha, Tokyo, 1970.
3. Gross & Harris, Queueing Theory, John Wiley
4. Johnson L.A., Montgomery, Operations Research in Production Planning, Scheduling & Inventory Control, John Wiley, 1974.

## **OPERATIONS RESEARCH II**

**Queuing Theory:**

Basic Structures of queuing models, Poisson queues M/M/1, M/M/C for finite and infinite queue length, Non-Poisson queue -M/G/1, Machine-Maintenance (steady state).

**Reliability:**

Concept, Reliability Definition, System Reliability, System Failure rate, Reliability of the Systems connected in Series or / and parallel.

**Information Theory:**

Introduction, Communication Processes memory less channel, the channel matrix, Probability relation in a channel, noiseless channel. A Measure of information- Properties of Entropy function, Measure of Other information quantities marginal and joint entropies, conditional entropies, expected mutual information, Axiom for an Entropy function, properties of Entropy function. Channel capacity, efficiency and redundancy. Encoding Objectives of Encoding. Shannon Fano Encoding Procedure, Necessary and sufficient Condition for Noiseless Encoding.

**Simulation:**

Introduction, Steps of simulation process, Advantages and disadvantages of simulation, Stochastic simulation and random numbers Monte Carlo simulation, Random number, Generation, Simulation of Inventory Problems, Simulation of Queuing problems, Role of computers in Simulation, Applications of Simulations.

**References:**

1. Ronald, V. Hartley, Operations Research A Managerial Emphasis Goodyear Publishing Company Inc., 1976, California.

2. Beveridge & Schechter, Optimization Theory and Practice, McGraw Hill Kogakusha, Tokyo, 1970.
3. Gross and Harris, Queueing Theory, John Wiley
4. Johnson L.A., Montgomery, Operations Research in Production Planning, Scheduling & Inventory Control, John Wiley, 1974.

## QUANTUM MECHANICS I

### **Origin of the Quantum Theory :**

Black-body radiation, Inadequacy of classical theory, The old quantum theory, Bohr-Sommerfeld theory, Atomic Spectra, Photoelectric effect and Compton effect, Matter waves, Wave-particle duality, Electron diffraction experiment.

### **Basic Concepts :**

Wave function of a free particle, Uncertainty and Complementarity principles, Gedanken experiments, wave packet, Schrödinger wave equation, Statistical interpretation of the wave function, Formal solution of the Schrödinger equation.

### **Simple Applications (exact solutions) :**

One dimensional potential step, Potential barrier, Square-well potential, Linear harmonic oscillator, Three-dimensional box potential, Spherically symmetric potential, Hydrogen atom bound-state problems.

### **Dynamical Variables and Operators :**

Operators corresponding to physical observables, Expectation values of observables, The virial theorem, Eigenfunction and eigenvalues of operators, Discrete and continuous spectra, Commutativity of operators, Heisenberg's uncertainty relations, The minimum uncertainty product, Heisenberg's equation of motion for operators.

### **Transformation Theory :**

Adjoint operator, Hermitian operator, Projection operator, Degeneracy, Unitary transformation, Matrix representation of wave functions and operators, Change of basis, Transformation of matrix elements, Dirac's Bra and Ket notation, Completeness and normalization of eigen functions, Common set of eigen functions of two operators, Compatibility of observables.

### **Symmetries and Invariance :**

Angular momentum eigenvalues and eigenfunctions, Spin, Addition of two angular momenta, Rotation groups, Identical particles, Pauli exclusion principle, Invariance and conservation theorems.

### **References :**

1. Heisenberg, The Physical Principles of the Quantum Theory , Dover Pub.,1930
2. Dirac, P. A. M., The Principles of Quantum Mechanics, Oxford University Press, 1981.
3. Mandl, F., Quantum Mechanics , Butterworths Sci. Pub., London, 1957.
4. Mathews, . P. T., Introduction to Quantum Mechanics, McGraw Hill, 1963.

5. Schiff, L. I., Quantum Mechanics, McGraw Hill, 1968.
6. Messiah, A. Quantum Mechanics, Vol. I & II, North – Holland Pub. Co., 1962.
7. Bransden, B. H. & Joachain, C. J., Introduction to Quantum Mechanics, Oxford University Press, 1989..
8. Burke, P. G., Potential Scattering in Atomic Physics , Plenum Press, New York, 1977.
9. Joachain, C. J., Quantum Collision Theory, North-Holland Pub. Co., 1975.
10. Bransden, B. H., Atomic Collision Theory, W. A. Benjamin Inc., N. Y., 1970.
11. Geltman, S., Topics in Atomic Collision Theory, Academic Press. 1969.
12. Wu, T. Y. & Olmura, T., Quantum Theory of Scattering, Prentice Hall, New Jersey, 1962.
13. Mott N. F. & Massey, H. S. W., Theory of Atomic collisions, Clarendon Press, Oxford, 1965.
14. Goldberger M. L. & Watson, K. M. , Collision Theory, Wiley, N. Y., 1964.
15. Newton, R. G., Scattering Theory of Waves and Particles , McGraw – Hill, 1966.

## QUANTUM MECHANICS II

### **Relativistic Kinematics :**

Klein-Gordon equation, Dirac equation for a free particle and its Lorentz covariance, Hole theory and positron, Electron spin and magnetic moment. Approximation Methods (time-independent) Rayleigh-Schrödinger perturbation method, An harmonic oscillator, Stark effect in hydrogen atom, Zeeman effect, Ground state energy of helium atom.

### **Elements of Second Quantization of a System :**

Creation and Annihilation operator, Commutation and Anti-commutation rules, Relation with Statistics - Bosons and Fermions.

### **Collision Theory :**

Basic concepts, Cross sections, Laboratory and center-of-mass coordinates, Rutherford scattering, Quantum mechanical formulation – time independent and time-dependent, Scattering of a particle by a short-range potential, Scattering by Coulomb potential, Scattering by screened Coulomb field, Scattering by complex potential.

### **Integral Equation Formulation:**

Lippmann-Schwinger integral equation and its formal solutions, Integral representation of the scattering amplitude, Convergence of the Born Series, Validity of Born approximation, Transition probabilities and cross sections. Semi-Classical Approximations : WKB approximation, Eikonal approximation.

### **Variational Principles in the Theory of Collisions :**

General formulation of the variational principle, Hulthen, Kohn-Hulthen and Schwinger variational methods, Determination of Phase shifts, Scattering length and scattering amplitude for central force problems, Bound (minimum) principles.

**References :**

1. Messiah, A. Quantum Mechanics, Vol. I & II, North – Holland Pub. Co., 1962.
2. Bransden, B. H. & Joachain, C. J., Introduction to Quantum Mechanics, Oxford University Press, 1989..
3. Burke, P. G., Potential Scattering in Atomic Physics , Plenum Press, New York, 1977.
4. Joachain, C. J., Quantum Collision Theory, North-Holland Pub. Co., 1975.
5. Bransden, B. H., Atomic Collision Theory, W. A. Benjamin Inc., N. Y., 1970.
6. Geltman, S., Topics in Atomic Collision Theory, Academic Press. 1969.
7. Wu, T. Y. and Olmura, T., Quantum Theory of Scattering, Prentice Hall, New Jersey, 1962.
8. Mott N. F. & Massey, H. S. W., Theory of Atomic collisions Clarendon Press, Oxford, 1965.
9. Goldberger M. L. & Watson, K. M. , Collision Theory, Wiley, N. Y., 1964.
10. Newton, R. G., Scattering Theory of Waves and Particles , McGraw Hill, 1966.

**COMPUTATIONAL FLUID DYNAMICS-I****Finite Difference methods:**

Solution of O.D.E., The method of factoriazation, iterative methods, upwind corrected schemes, Hermitian method. Solution of a one-dimensional linear parabolic equation; Noncentered schemes, Leapfrog Dufort-Frankel scheme, Solution of one-dimensional Non-linear parabolic and hyperbolic equations, Explicit and Implicit methods, The ADI method, Explicit splitting method for two dimensional equation.

**Finite Element Methods:**

Variational formulation of operator equations and Galerkin's method, the construction of the finite elements, convergence rates for F. E. M., Stability of F. E. M., Elementary ideas of Finite volume method, Spectral method. Some simple applications of Fluid Dynamics Problems.

**References:**

1. Linz, P., Theoretical Numerical Analysis, An Introduction To Advance Technique, John Wiley & Sons.
2. Peyret, R. & Taylor, T. D., Computational Methods for fluid Flow, Springer – Verlag.
3. Wesseling, P., Principles of Computational Fluid Dynamics, Springer-Verlag, 2000 .
4. Anderson, J. D., Computational Fluid Dynamics : The Basics With Applications, McGraw-Hill, 1998.
5. Fletcher, C. A. J., Computational Techniques for Fluid Dynamics, Vol. I and II, Springer-Verlag.
6. Anderson, D. A. , Tanehill, J. C. & Pletcher, R. H., Computational Fluid Mechanics and Heat Transfer, Hemisphere Publishing Corporation.

## COMPUTATIONAL FLUID DYNAMICS-II

Multigrid method, Conjugate – Gradient method. Incompressible Navier – Stokes (NS) equations – Boundary conditions, Spatial and temporal discretization on collocated and on staggered grids. Development of the MAC Method for NS equations, Implementation of boundary conditions.

Grid Generation by Algebraic mapping : One-dimensional stretching functions, Boundary – Filtered Coordinate Systems : Elliptic Grid generation. Solution of Euler Equations in General Co-ordinates. Numerical Solution of NS Equations in General Co-ordinates.

### References :

1. Linz, P., Theoretical Numerical Analysis, An Introduction To Advance Technique, John Wiley & Sons.
2. Peyret, R. and Taylor, T. D., Computational Methods for fluid Flow, Springer – Verlag.
3. Wesseling, P., Principles of Computational Fluid Dynamics, Springer-Verlag, 2000 .
4. Anderson, J. D., Computational Fluid Dynamics : The Basics With Applications, McGraw-Hill, 1998.
5. Fletcher, C. A. J., Computational Techniques for Fluid Dynamics, Vol. I and II, Springer-Verlag.
6. Anderson, D. A. , Tanehill, J. C. & Pletcher, R. H., Computational Fluid Mechanics and Heat Transfer, Hemisphere Publishing Corporation.

### Pure Mathematics branches (P):

#### ADVANCED COMPLEX ANALYSIS I

The Functions  $M(r)$ ,  $A(r)$ , Hadamard Theorem on Growth of  $\log M(r)$ , Schwarz Inequality, Borel-Caratheodory Inequality.

Entire Functions, Growth of an Entire Function, Order and Type and their Representations in terms of the Taylor Coefficients, Distribution of Zeros, Dirichlet Series, Schottky's Theorem (no proof), Picard's Little Theorem, Weierstrass Factor Theorem, The Exponent of Convergence of Zeros, Hadamard Factorization Theorem, Canonical Product, Borel's First Theorem, Borel's Second Theorem (statement only).

Analytic Continuation, Natural Boundary, Analytic Element, Global Analytic Function, Multiple Valued functions, Branch Points and Branch Cut, Riemann Surfaces.

### References:

1. Ahlfors, L.V., Complex Analysis, McGraw-Hill, 1979.
2. Markusevich, A.I., Theory of Functions of a Complex Variable, Vol. I, II, III, AMS Chelsea Publishing.
3. Copson, E.T., An Introduction to the Theory of Functions of a Complex Variable, Oxford University Press, London.
4. Rudin, W., Real and Complex Analysis, McGraw-Hill Book Co., 1966.
5. Hille, E., Analytic Function Theory (2 vols.), Gonn & Co., 1959.
6. Titchmarsh, E.C., The Theory of Functions, Oxford University Press, London.

7. Hayman, W.K., Meromorphic Functions, Oxford : Clarendon press 1964.
8. Kaplan, W., An Introduction to Analytic Functions, Addison Wesley series in Mathematics, 1966.
9. Levin, B. Ja., Distribution of zeros of entire functions, Translations of mathematical monographs, Vol. 5, American Mathematical Society.

## ADVANCED COMPLEX ANALYSIS II

Meromorphic Functions, Expansions, Definition of the functions  $m(r, a)$ ,  $N(r, a)$  and  $T(r, a)$ . Nevanlinna's First Fundamental Theorem, Cartan's Identity and Convexity Theorems, Order of Growth, Order of a Meromorphic Function, Comparative Growth of  $\log M(r)$  and  $T(r)$ , Nevanlinna's Second Fundamental Theorem, Estimation of  $S(r)$  (statement only), Nevanlinna's Theory of Deficient Values, Milloux Theorem, Five point uniqueness theorem, A brief review of uniqueness theory of entire and meromorphic functions via shared values and sets.

### References:

1. Yang C. C. and Yi H. X., Uniqueness Theory of Meromorphic Function, Springer Netherlands, 2003
2. Ahlfors, L.V., Complex Analysis, McGraw-Hill, 1979.
3. Markusevich, A.I., Theory of Functions of a Complex Variable, Vol. I, II, III, AMS Chelsea Publishing.
4. Copson, E.T., An Introduction to the Theory of Functions of a Complex Variable, Oxford University Press, London.
5. Rudin, W., Real and Complex Analysis, McGraw-Hill Book Co., 1966.
6. Hille, E., Analytic Function Theory (2 vols.), Gonn & Co., 1959.
7. Titchmarsh, E.C., The Theory of Functions, Oxford University Press, London.
8. Hayman, W.K., Meromorphic Functions, Oxford : Clarendon press 1964.
9. Kaplan, W., An Introduction to Analytic Functions, An Introduction to Analytic Functions, Addison Wesley series in Mathematics, 1966.

## DIFFERENTIAL GEOMETRY I

### Calculus on Euclidean Spaces:

Euclidean Spaces, Tangent Vectors, Directional Derivatives, Curves in  $\mathbb{R}^3$ , 1-forms, Differential Forms, Mappings.

### Frame Fields:

Dot Product, Curves, Frenet Formulae, Arbitrary Speed Curves, Covariant Derivatives, Frame Fields, Connection Forms, The Structural Equations.

### Euclidean Geometry:

Isometries of  $\mathbb{R}^3$ , Tangent Map of an Isometry, Orientation, Euclidean Geometry, Congruence of Curves.

### **Calculus on a Surface:**

Surfaces in  $\mathbb{R}^3$ , Patch Computations, Differentiable Functions and Tangent Vectors, Differentiable Forms on a Surface, Mappings of Surfaces, Integration of Forms, Topological Properties of Surfaces, Manifolds.

### **References:**

1. O'Neill, Barret, Elementary Differential Geometry, Elsevier Academic Press, 2006.
2. Pressley, A., Elementary Differential Geometry, Springer, 2004.

## **DIFFERENTIAL GEOMETRY II**

### **Tensor Algebra:**

Finite Dimensional Real Linear Spaces, Their Subspaces and Dual Spaces. Summation Convention, Change of Bases, Contravariant and Covariant Vectors. Multilinear Functionals, Tensor Spaces, Algebra of Tensors. Symmetric and Skew-Symmetric Tensors. Exterior Algebra.

### **Manifolds:**

Definition and Examples. Differentiable Curves. Submanifolds. Tangents. Differential of a Map.

### **Vector Analysis on Manifolds:**

Vector and Tensor Fields, Integral Curves and Flows, Lie Bracket. One Parameter Group of Transformations. Exponential Maps.

### **Linear Connections:**

Linear Connections, Their Torsion and Curvature. Riemannian Manifolds Riemannian Manifolds. Curvature Tensor, Ricci Tensor, Scalar Curvature, Sectional Curvature.

### **References:**

1. Bishop, Richard L. & Goldberg, Samuel I., Tensor Analysis on Manifolds, Macmillan, 1968.
2. Hicks, Noel J., Notes on Differential Geometry, Van Nostrand. 1965.
3. Kumaresan, S., Differential Geometry and Lie Groups, Book Agency, 2002.
4. Boothby, William M., An Introduction to Differentiable Manifolds and Riemannian Geometry, 1975.

## **ADVANCED REAL ANALYSIS I**

### **Ordinal Numbers:**

Order type, well ordered sets, transfinite induction, ordinal numbers, comparability of ordinal numbers, Arithmetic of ordinal numbers. First uncountable ordinal ( $\Omega$ ).

### **Descriptive properties of sets:**

Perfect sets, every closed set is the union of a perfect set and a finite or denumerable set. Nowhere dense set. First category, second category and residual sets. In a complete metric space  $X$  every subset of  $X$  is residual in  $X$  if and only if it contains a dense  $G_\delta$ -set. Borel sets of order type  $\alpha$  ( $< \Omega$ ) and its properties.

**Functions of special classes:**

Baire class functions of order type  $\alpha$  ( $< \Omega$ ) and its properties. Relation Between Baire functions and Borel sets.

**Continuity:**

Lower and upper semi-continuous functions with their properties. Absolute continuity and Lusin (N) condition.

Lebesgue density point of a set and Lebesgue density theorem, Approximate continuity and its simple properties.

**Derivative:**

The Vitali-covering theorem, Dini's derivatives and its properties. Derivative of a monotone function, Determining a function by its derivative. Lebesgue point.

**References:**

1. Bruckner, A.M., Bruckner, J.B. and Thomson, B.S., Real Analysis, Prentice Hall.
2. Goffman, C., Real Functions, Rinehart.
3. Jeffrey, R.L., The Theory of Functions of a Real Variable, University of Toronto Press.
4. Natanson, I.P., Theory of Functions of a Real Variable, Vol. I & II, Frederick Ungar Publishing.
5. Hobson, E.W., Theory of Functions of a Real Variable, Vol. I & II., Dover.
6. Royden, H.L., Real Analysis, MacMillan.
7. Munroe, M.E., Introduction to Measure and Integration, Addison Wesley.
8. Lee, P.Y., Lanzhou Lectures on Henstock Integration, world Scientific.
9. Das, A.G., The Riemann, Lebesgue and Generalized Riemann Integral, Narosa.

## ADVANCED REAL ANALYSIS II

**Fourier Series:**

Trigonometric series, Fourier series, Dirichlet's kernel, pointwise convergence-Dini's test, Jordan test, convergence of Cesaro means-Fejer's theorem, Lebesgue-Fejer's theorem, Riemann's theorem. Cantor's uniqueness theorem.

**Integration on  $\mathbb{R}$ :**

Henstock integral: Gauge functions, finite partition, Cousin lemma, definition of Henstock integral and examples, Saks-Henstock lemma, Linearity property, Fundamental theorem. Relation of Henstock integral with Newton, Riemann and Lebesgue integrals. Absolute integrability of Henstock integral, Monotone and Dominated Convergence theorem of Henstock integral.

**General Measure and Integration:**

Measure space, measurable functions, integration of non-negative function, convergence theorems, Fatou's lemma, Signed measure, positive and negative sets. Hahn and Jordan decomposition theorems. Absolute continuous and singular measures, Radon-Nikodym theorem and its consequences.

**References:**

1. Bruckner, A.M., Bruckner, J.B. and Thomson, B.S., Real Analysis, Prentice Hall.
2. Goffman, C., Real Functions, Rinehart.
3. Jeffrey, R.L., The Theory of Functions of a Real Variable, University of Toronto Press.
4. Natanson, I.P., Theory of Functions of a Real Variable, Vol. I & II, Frederick Ungar Publishing.
5. Hobson, E.W., Theory of Functions of a Real Variable, Vol. I & II., Dover.
6. Royden, H.L., Real Analysis, MacMillan.
7. Munroe, M.E., Introduction to Measure and Integration, Addison Wesley.
8. Lee, P.Y., Lanzhou Lectures on Henstock Integration, world Scientific.
9. Das, A.G., The Riemann, Lebesgue and Generalized Riemann Integral, Narosa.

**ADVANCED TOPOLOGY I**

Compactness, Limit point compactness, sequentially compact spaces, countably compact spaces. Locally compact spaces.

Countability Axioms, The Separation Axioms, Lindelf spaces, Regular spaces, Normal spaces, Urysohn Lemma, Tietze Extension Theorem.

Tychonoff Theorem & Compactification : Tychonoff Theorem, Completely Regular spaces, Local Compactness, One-point compactification, Stone-Čech Compactification.

Metrization: Urysohn Metrization Theorem, Topological Imbedding, Imbedding Theorem of a regular space with countable base in  $R_n$ , Partitions of Unity, Topological  $m$ -Manifolds, Imbedding Theorem of a compact  $m$ -manifold in  $R_n$ .

Local Finiteness, Nagata-Smirnov Metrization Theorem, Paracompactness, Stone's Theorem, Local Metrizability, Smirnov Metrization Theorem. Uniform Spaces.

**References:**

1. Munkres, J.R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. Dugundji, J., Topology, Allyn and Bacon, 1966.
3. Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
4. Kelley, J. L., General Topology, Van Nostrand Reinhold Co., New York, 1995.
5. Hocking, J. & Young, G., Topology, Addison-Wesley Reading, 1961.
6. Steen, L. & Seebach, J., Counter Examples in Topology, Holt, Reinhart and Winston, New York, 1970.

**ADVANCED TOPOLOGY II**

Nets and Filters : Directed Sets, Nets and Sub-nets, Convergence of a net, Ultranets, Partially Ordered Sets and Filters, Convergence of a filter, Ultra filters, Basis and Subbase of a filter, Nets and Filters in Topology.

Complete Metric Spaces & Function Spaces: Complete Metric Spaces, Baire Category Theorem, The Peano Space-Filling Curve, Hahn-Mazurkiewicz Theorem (statement only). Compactness in Metric Spaces, Equicontinuity.

Pointwise and Compact Convergence, The Compact-Open Topology, Stone-Weierstrass Theorem, Ascoli's Theorem, Baire Spaces, A nowhere Differentiable Function.

An Introduction to Dimension Theory, Topological notion of Lebesgue dimension.

**References:**

1. Munkres, J.R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. Dugundji, J., Topology, Allyn and Bacon, 1966.
3. Simmons, G.F., Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
4. Kelley, J. L., General Topology, Van Nostrand Reinhold Co., New York, 1955.
5. Hocking, J. & Young, G., Topology, Addison-Wesley Reading, 1961.
6. Steen, L. & Seebach, J., Counter Examples in Topology, Holt, Reinhart and Winston, New York, 1970.

## OPERATOR THEORY I

**Bounded linear Operators:**

Resolvent set, Spectrum, Point spectrum, Continuous spectrum, Residual spectrum, Approximate point spectrum, Spectral radius, Spectral properties of a bounded linear operator, Spectral mapping theorem for polynomials.

**Banach Algebra:**

Definition of normed and Banach Algebra and examples, Singular and Non-singular elements, The spectrum of an element, The spectral radius.

**Compact linear operators:**

Spectral properties of compact linear operators on a normed linear space, Operator equations involving compact linear operators, Fredholm alternative theorem, Fredholm alternative for integral equations. Spectral theorem for compact normal operators.

**References:**

1. Brown, A. & Percy, C., Introduction to Operator Theory, I, II, Springer-Verlag, 1977.
2. Coway, J.B., A Course in Functional Analysis, Springer, 1990.
3. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.
4. Rudin, W., Real and Complex Analysis, Tata McGraw Hill, 1974.
5. Kreyszig, E., Introductory Functional Analysis with Applications, John Wiley and sons.
6. Bachman, G. & Narici, L., Functional Analysis, Dover Publications.
7. Taylor, A. and Lay, D., Introduction to Functional Analysis, John Wiley and Sons.
8. Dunford, N. and Schwartz, J.T., Linear Operators 3, John Wiley and Sons.
9. Halmos, P.R., Introduction to Hilbert space and the theory of Spectral Multiplicity, Chelsea Publishing Co., N.Y.

## OPERATOR THEORY II

**Self-adjoint operators:**

Spectral properties of bounded self-adjoint linear operators on a complex Hilbert space, Positive operators, Square root of a positive operator, Projection operators, Spectral family of a bounded

self-adjoint linear operator and its properties, Spectral theorem for a bounded self-adjoint linear operator.

### **Normal Operators:**

Spectral properties for bounded normal operators, Spectral theorem for bounded normal operators.

### **Unbounded linear operators in Hilbert space:**

Hellinger-Toeplitz theorem, Symmetric and self-adjoint operators, Closed linear operators, Spectrum of an unbounded self-adjoint linear operator, Cayley Transformation of an operator, Spectral theorem for unitary and self-adjoint operators, Multiplication operator and differentiation operator, Application to Quantum Mechanics.

### **References:**

1. Brown, A. and Pearcy, C., Introduction to Operator Theory, I, II, Springer-Verlag, 1977.
2. Coway, J.B., A Course in Functional Analysis, Springer, 1990.
3. Rudin, W., Functional Analysis, Tata McGraw Hill, 1992.
4. Rudin, W., Real and Complex Analysis, Tata McGraw Hill, 1974.
5. Kreyszig, E., Introductory Functional Analysis with Applications, John Wiley and sons.
6. Bachman, G. and Narici, L., Functional Analysis, Dover Publications.
7. Taylor, A. and Lay, D., Introduction to Functional Analysis, John Wiley and Sons.
8. Dunford, N. and Schwartz, J.T., Linear Operators 3, John Wiley and Sons.
9. Halmos, P.R., Introduction to Hilbert space and the theory of Spectral Multiplicity, Chelsea Publishing Co., N.Y.

## **ADVANCED FUNCTIONAL ANALYSIS I**

Normed linear spaces, Banach spaces. Stone-Weierstrass Theorem, Ascoli-Arzelà Theorem.

Bounded linear operators. Dual of a normed linear space. Hahn-Banach theorem, Computing the dual of well known Banach spaces.

Weak and weak\* topologies, Banach-Alaoglu Theorem. The double dual.

$L^p$ -spaces, Completeness and other Properties. Riesz representation theorem for the space  $C[0; 1]$ .

Linear Topological Spaces, Locally Convex Spaces and their Characterization in terms of a family of Semi-norms.

### **References:**

1. Rudin, W., Real and complex analysis, McGraw-Hill, 1987.
2. Rudin, W., Functional analysis, McGraw-Hill, 1991.
3. Conway, J.B., A course in functional analysis, GTM (96), Springer-Verlag, 1990.
4. Yosida, K., Functional analysis, Springer-Verlag, 2004.
5. Katznelson, Y., An introduction to harmonic analysis, Dover Publications, 1976.
6. Stein, E.M. & Shakrachi, R., Fourier Analysis: An Introduction, Princeton Lectures in Analysis.

7. Hernez, E. Weiss, G., A first course on wavelets, Studies in Advanced Mathematics, CRC Press, 1996.
8. Kelley, J.L. & Namioka, I., Linear Topological Spaces, D.Van Nostrand Company, 1963.
9. Aliprantis, C.D. & Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd.,1998.
10. Goffman, C. & Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New Delhi,1987.
11. Taylor, A.E., Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.

## **ADVANCED FUNCTIONAL ANALYSIS II**

Krein-Milman Theorem and its Applications, Uniform Convexity, Strict Convexity and their Applications.

Fourier and Fourier-Stieltjes' series, summability kernels, convergence tests.

Fourier transforms, the Schwartz space, Fourier Inversion and Plancherel theorem. Maximal functions and boundedness of Hilbert transform. Statement of Paley-Wiener Theorem. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem (discussion without proof). Introduction to wavelets and multi-resolution analysis.

### **References:**

1. Rudin, W., Real and complex analysis, McGraw-Hill, 1987.
2. Rudin, W., Functional analysis, McGraw-Hill, 1991.
3. Conway, J.B., A course in functional analysis, GTM (96), Springer-Verlag, 1990.
4. Yosida, K., Functional analysis, Springer-Verlag, 2004.
5. Katznelson, Y., An introduction to harmonic analysis, Dover Publications, 1976.
6. Stein, E.M. & Shakrachi, R., Fourier Analysis: An Introduction, Princeton Lectures in Analysis.
7. Hernez, E. & Weiss, G., A first course on wavelets, Studies in Advanced Mathematics, CRC Press,1996.
8. Kelley, J.L. & Namioka, I., Linear Topological Spaces, D.Van Nostrand Company, 1963.
9. Aliprantis, C.D. & Burkinshaw, O., Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd.,1998.
10. Goffman, C. & Pedrick, G., First Course in Functional Analysis, Prentice Hall of India, New Delhi,1987.
11. Taylor, A.E., Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.