BISECTION METHOD

- ***Theory*:** The Bisection method is one of the simplest and most reliable of iterative methods for the solutions of nonlinear equations. This method, also known as *binary chopping* or *half-interval method*, relies on the fact that if f(x) is real and continuous in the interval a<x<b, and f(a) and f(b) are of opposite signs, that is,

  $$f(a)\ f(b)<0$$

  Then there is at least one real root between a and b. There may be more than one root in the interval.

  Let, x_1= a and x_2= b. Let us also define another point x_0 to be the middle point between a and b, that is,

  $$x_0 = \frac{x_1 + x_2}{2}$$

  Now, there exists the following three conditions:

1. If f(x_0)=0, we have a root at x_0.
2. If f(x_0) f(x_1)<0, then there is a root between x_0 and x_1.
3. If f(x_0) f(x_2)<0, then there is a root between x_0 and x_2.

It follows that by testing the sign of the function at midpoint, we can deduce which part of the interval contains the root. This is illustrated in Fig given below. It shows that, since f(x_0) and f(x_2) are of opposite sign, a root lies between x_0 and x_2. We can further divide this subinterval into two halves to locate a new subinterval containing the root. This process can be repeated until the interval containing the root is as small as we desire.

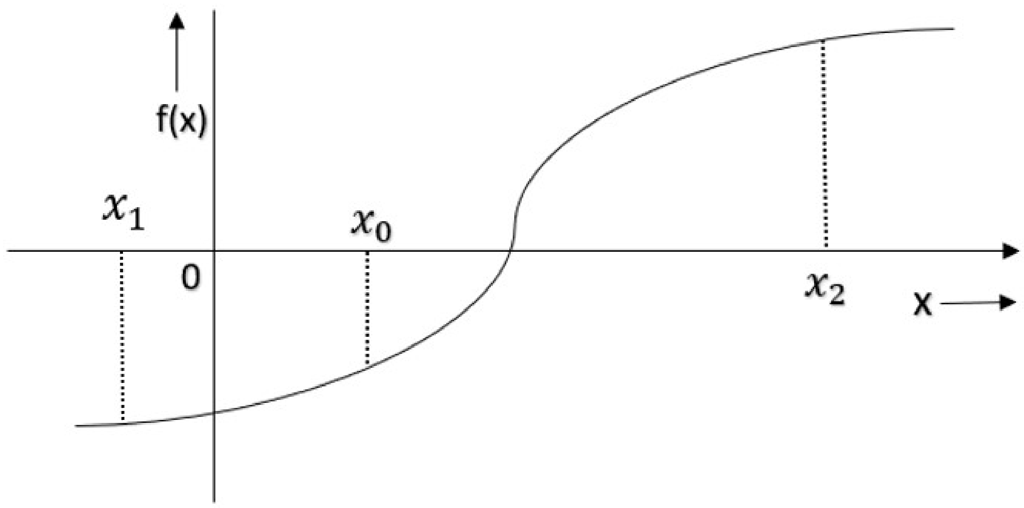

Fig: Illustration of bisection method

- ***ALGORITHM:***

1. Decide the order and enter co-efficients of polynomial in decreasing order
2. If $f(i) > 0$ and $f(i+1) < 0$ then $aa(k) = x(i)$ and $bb(k) = x(i+1)$
3. If $f(i) < 0$ and $f(i+1) > 0$ then $aa(k) = x(i)$ and $bb(k) = x(i+1)$
4. Compute $xx = \frac{aa(k)+bb(k)}{2}$
5. Compute $fx = f(x)$
6. If $f(x) * f(a) < 0$ then
   Set $bb(k) = xx$

   Else

   Set $aa(k) = xx$
7. If absolute value of $f(x)$ is less than 0.0001, then the root is xx
   Write the value of xx
   Go to step 8,
   Else
   Go to step 4
8. Stop

```
!	BISECTION METHOD
	dimension a(100)
	real x(10000), f(10000), aa(20), bb(20)
	real xx, fx, fa, fb

	write(*,*) 'enter the order of the polynomial'
	read(*,*) n
	write(*,*) 'enter the co-efficient of the polynomial in decreasing order'
	do i=1, n+1
	read(*,*) a(i)

!	write(*,*) a(i)
	enddo
	h=0.02
	x(1)=-4
	do i=1,1000
	call cal(n,a,x(i),f(i))
	x(i+1)=x(i)+h
	enddo

	k=1
	do i=1,1000

	if((f(i).gt.0.0).and.(f(i+1).lt.0.0)) then
	aa(k)=x(i)
	bb(k)=x(i+1)
	k=k+1
```

```
        elseif((f(i).lt.0.0).and.(f(i+1).gt.0.0)) then
        aa(k)=x(i)
        bb(k)=x(i+1)
        k=k+1

        endif
        enddo
        m=k-1
        write(*,*) 'the number of roots is', m

!::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
        do k=1,m
20      xx=(aa(k)+bb(k))/2

        call cal(n,a,aa(k),fa)
        call cal(n,a,bb(k),fb)
        call cal(n,a,xx,fx)

        if(abs(fx).lt.0.0001) goto 30
        if((fx*fa).lt.0.0) then
        bb(k)=xx
        goto 20
        elseif(fx*fb.lt.0.0) then
        aa(k)=xx
        goto 20
        endif
```

```
30      write(*,*) "the",k," root is", xx
        enddo

!:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
        stop
        end
!:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
        subroutine cal(n,a,x,fx)
        real a(n+1), x, fx, sum

        sum=0
        do i=1,n+1
        sum=sum+a(i)*x**(n-i+1)

        enddo

        fx=sum
        return
        end
```

- ***Output:***

```
enter the order of the polynomial
3
enter the co-efficient of the polynomial in decreasing order
1
-2
-5
6
the number of roots is         3
the         1 root is     -1.999997
the         2 root is      1.000016
the         3 root is      3.000004
Stop - Program terminated.

Press any key to continue
```

- ***ADVANTAGES OF BISECTION METHOD:***

  1. The Bisection method is always convergent. Since the method brackets the root, the method is guaranteed to converge.

  2. As iterations are conducted, the interval gets halved. So one can guarantee the error in the solution 0f the equation.

- ***DISADVANTAGES OF BISECTION METHOD:***

  Biggest dis-advantage is the slow convergence rate. Typically bisection is used to get an initial estimate for such faster methods such as Newton-Raphson that requires an initial estimate. There is also the inability to detect multiple roots.